Collins

CAMBRIDGE IGCSE™ MATHS STUDENT'S BOOK ANSWERS

Answers to Chapter 1

1.1 Square numbers and cube numbers

Exercise 1A

1 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400

2 1 + 3 = 4, 1 + 3 + 5 = 9, 1 + 3 + 5 + 7 = 16, 1 + 3 + 5 + 7 + 9 = 25, 1 + 3 + 5 + 7 + 9 + 11 = 36, 1 + 3 + 5 + 7 + 9 + 11 + 13 = 49

3 **a** 50, 65, 82 **b** 98, 128, 162
c 51, 66, 83 **d** 48, 63, 80

4 **a** 25, 169, 625, 1681
b Answers in each row are the same

5 **a** 13 **b** 5 **c** 9 **d** 11 **e** 20

6 36 and 49

Exercise 1B

1 **a** 12, 24, 36 **b** 20, 40, 60 **c** 15, 30, 45
d 18, 36, 54 **e** 35, 70, 105

2

	Square number	Factor of 70
Even number	16	10
Multiple of 7	49	35

3 4761 (69^2) or 1764 (42^2)

4 24 seconds

5 30 seconds

6 **a** 12 **b** 9 **c** 6 **d** 13
e 15 **f** 16 **g** 10 **h** 17

Exercise 1C

1 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, 1331, 1728

2 9, 36, 100: square numbers
1 + 8 + 27 + 64 + 125 = 225
1 + 8 + 27 + 64 + 125 + 216 = 441
1 + 8 + 27 + 64 + 125 + 216 + 343 = 784

3 **a** 126 217 344
b 124 215 342
c 250 432 686
d 216 125 64

4 **a** 153, 370, 371
b Each answer is the sum of the cubes of its digits

5 1729

6 $69^2 = 4761$ and $69^3 = 328509$ The answers use all the digits from 0 to 9 exactly once.

1.2 Multiples of whole numbers

Exercise 1D

1 **a** 3, 6, 9, 12, 15 **b** 7, 14, 21, 28, 35
c 9, 18, 27, 36, 45 **d** 11, 22, 33, 44, 55
e 16, 32, 48, 64, 80

2 **a** 72, 132, 216 **b** 161, 91 **c** 72, 102, 132, 78, 216

3 **a** 98 **b** 99 **c** 96 **d** 95 **e** 98 **f** 96

4 4 or 5 (as 2, 10 and 20 are not realistic answers)

5 **a** 18 **b** 28 **c** 15

6 5 numbers: 18, 36, 54, 72, 90

1.3 Factors of whole numbers

Exercise 1E

1 **a** 1, 2, 5, 10 **b** 1, 2, 4, 7, 14, 28
c 1, 2, 3, 6, 9, 18 **d** 1, 17
e 1, 5, 25 **f** 1, 2, 4, 5, 8, 10, 20, 40
g 1, 2, 3, 5, 6, 10, 15, 30 **h** 1, 3, 5, 9, 15, 45
i 1, 2, 3, 4, 6, 8, 12, 24 **j** 1, 2, 4, 8, 16

2 **a** 55 **b** 67 **c** 29
d 39 **e** 65 **f** 80

3 **a** 2 **b** 2 **c** 3 **d** 5 **e** 3
f 3 **g** 7 **h** 5 **i** 10 **j** 11

4 5

Exercise 1F

1 **a** 168 **b** 105 **c** 84 **d** 84
e 96 **f** 54 **g** 75 **h** 144

2 **a** 8 **b** 7 **c** 4 **d** 14
e 4 **f** 9 **g** 5 **h** 4
i 3 **j** 16 **k** 5 **l** 18

3 18 and 24

1.4 Prime numbers

Exercise 1G

1 23 and 29

2 97

3 All these numbers are not prime.

4 3, 5, 7

5 Only if all 31 bars are in a single row, as 31 is a prime number and its only factors are 1 and 31.

1.5 Prime factorisation

Exercise 1H

1 **a** 36 **b** 105 **c** 250 **d** 816
e 714 **f** 1715 **g** 1089 **h** 1352

2 **a** $2 \times 3^2 \times 5$ **b** $2^3 \times 19$ **c** 2^6
d $2 \times 3 \times 5 \times 11$ **e** 17^2 **f** $2^5 \times 5^2$
g It is a prime number and cannot be factorised.
h $7 \times 11 \times 13$

3 $77 = 7 \times 11$; $129 = 3 \times 43$; $221 = 13 \times 17$

4 **a** $900 = 2^2 \times 3^2 \times 5^2$ **b** $1800 = 2^3 \times 3^2 \times 5^2$
c $1350 = 2 \times 3^3 \times 5^2$

5 144 — $2^2 \times 3^4$; 200 — $2^4 \times 3^2$; 324 — $2^3 \times 5^2$; 500 — $2^2 \times 5^3$

6 **a** $2 \times 3 \times 5 \times 7 = 210$

b The answer to **a** $\times 11 = 2 \times 3 \times 5 \times 7 \times 11 = 2310$

7 **a** $2^2 \times 3^2 \times 17$ **b** $2 \times 3^2 \times 17$ **c** $2^3 \times 3 \times 17$

8 71, 73 and 79 because they are prime numbers

9 $456\,533 = 7^3 \times 11^3$

1.6 Finding the HCF and LCM

Exercise 1I

1 **a** $2 \times 3^2 = 18$ **b** $2^3 \times 3^4 = 648$

2 **a** 35 **b** 735

3 **a** $2^4 \times 3 \times 5$ **b** $2 \times 3^2 \times 7$ **c** 6 **d** 5040

4 **a** $2^3 \times 3^2$ and $2^2 \times 3^3$ **b** 36 **c** 216

5 **a** 16 **b** 576

6 **a** 33 **b** 2772

7 **a** 1 **b** 12 600

8 **a** $72 \times 162 = 11664$ **b** $18 \times 648 = 11664$

c You could do a similar calculation for the numbers in questions 2 to 6. You should find that the two products are equal each time.

1.7 Rational and irrational numbers

Exercise 1J

1 **a** yes **b** no **c** yes **d** yes **e** no
f yes **g** no **h** yes **i** yes

2 **a** rational **b** rational **c** irrational
d rational **e** irrational **f** rational
g rational **h** irrational **i** irrational

3 **a** $\frac{1}{300}$ **b** $\frac{10}{3}$ or $3\frac{1}{3}$ **c** $\frac{4}{17}$ **d** 0.4

4 5 (its reciprocal is not listed)

5 $\frac{5}{6}$

6 **a** 2.5 and 3.5 is one possible answer

b 0.4×2.5 is one possible answer

c Not possible

7 $\sqrt{2} \times \sqrt{8}$ is a possible answer

8 π and $4 - \pi$ is a possible answer

9 There are many possible answers. You could just give the same answer as question 8.

Answers to Chapter 2

2.1 Equivalent fractions

Exercise 2A

1 **a** $\frac{8}{20}$ **b** $\frac{3}{12}$ **c** $\frac{15}{40}$
d $\times 6$, $\frac{12}{18}$ **e** $\times 3$, $\frac{9}{12}$ **f** $\times 5$, $\frac{25}{40}$

2 **a** $\frac{2}{3}$ **b** $\frac{4}{5}$ **c** $\frac{5}{7}$ **d** $\div 6$, $\frac{2}{3}$
e $25 \div 5$, $\frac{3}{5}$ **f** $30 \div 3$, $\frac{7}{10}$

3 **a** $\frac{2}{3}$ **b** $\frac{1}{3}$ **c** $\frac{2}{3}$ **d** $\frac{3}{4}$ **e** $\frac{1}{3}$
f $\frac{1}{2}$ **g** $\frac{7}{8}$ **h** $\frac{4}{5}$ **i** $\frac{1}{2}$ **j** $\frac{1}{4}$

4 **a** $\frac{1}{2}, \frac{2}{3}, \frac{5}{6}$ **b** $\frac{1}{2}, \frac{5}{8}, \frac{3}{4}$ **c** $\frac{2}{5}, \frac{1}{2}, \frac{7}{10}$
d $\frac{7}{12}, \frac{2}{3}, \frac{3}{4}$ **e** $\frac{1}{6}, \frac{1}{4}, \frac{1}{3}$ **f** $\frac{3}{4}, \frac{4}{5}, \frac{9}{10}$

5 **a** $\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$

Explanations may involve ruling out other combinations

b $\frac{1}{2}$ as the smallest denominator is the biggest unit fraction

Diagrams may be used but must be based on equal sized area.

6 **a** $2\frac{1}{3}$ **b** $2\frac{2}{3}$ **c** $2\frac{1}{4}$
d $1\frac{3}{7}$ **e** $2\frac{2}{5}$ **f** $1\frac{2}{5}$

7 **a** $\frac{10}{3}$ **b** $\frac{35}{6}$ **c** $\frac{9}{5}$ **d** $\frac{37}{7}$ **e** $\frac{41}{10}$ **f** $\frac{17}{3}$
g $\frac{5}{2}$ **h** $\frac{13}{4}$ **i** $\frac{43}{6}$ **j** $\frac{29}{8}$ **k** $\frac{19}{3}$ **l** $\frac{89}{9}$

8 Students check their own answers.

9 $\frac{27}{4} = 6\frac{3}{4}$, $\frac{31}{5} = 6\frac{1}{5}$, $\frac{13}{2} = 6\frac{1}{2}$, so $\frac{27}{4}$ is the biggest since $\frac{1}{5}$ is less than $\frac{1}{2}$ and $\frac{3}{4}$ is greater than $\frac{1}{2}$

10 Any mixed number which is between 7.7272 and 7.9.
For example $7\frac{4}{5}$

2.2 Fractions and decimals

Exercise 2B

1 **a** $\frac{7}{10}$ **b** $\frac{2}{5}$ **c** $\frac{1}{2}$ **d** $\frac{3}{100}$ **e** $\frac{3}{50}$
f $\frac{13}{100}$ **g** $\frac{1}{4}$ **h** $\frac{19}{50}$ **i** $\frac{11}{20}$ **j** $\frac{16}{25}$

2 **a** 0.5 **b** 0.75 **c** 0.6 **d** 0.5333... **e** 3.66666...
f 0.625 **g** 0.636363... **h** 0.35 **i** 0.636 **j** 0.4444...

3 **a** 0.3, $\frac{1}{2}$, 0.6 **b** 0.3, $\frac{2}{5}$, 0.8 **c** 0.15, $\frac{1}{4}$, 0.35
d $\frac{7}{10}$, 0.71, 0.72 **e** 0.7, $\frac{3}{4}$, 0.8 **f** $\frac{1}{20}$, 0.08, 0.1
g 0.4, $\frac{1}{2}$, 0.55 **h** 1.2, 1.23, $1\frac{1}{4}$

4 **a** 1.66666... **b** 0.71428... **c** 0,55555... **d** 0.45454...

5 Store A – $\frac{1}{3}$ (0.33) is greater than $\frac{1}{4}$ (0.25)

6 **a** $\frac{12}{30} = \frac{2}{5}$ **b** 0.4

7 $\frac{7}{8}$ (= 0.875)

8 $\frac{2}{3}$ (= 0.67)

2.3 Recurring decimals

Exercise 2C

1 **a** 0.333… or $0.\dot{3}$ **b** 0.75 **c** 0.8333… or $0.8\dot{3}$
d 0.222… or $0.\dot{2}$ **e** 0.65 **f** 0.8181… or $0.\dot{8}\dot{1}$
g 0.1875 **h** 0.916 66… or $0.916\dot{6}$

2 **a** 0.4666… or $0.46\dot{6}$ **b** 0.9333… or $0.9\dot{3}$

3 **a** 0.1111… **b** 0.1666… **c** 0.2777… **d** 0.0555…

4 $\frac{8}{9}$

5 $\frac{8}{33}$

6 $\frac{11}{30}$

7 $\frac{1}{12}$

8 $2\frac{7}{15}$

9 $0.\dot{2}30769\dot{}$

10 **a** $0.0\dot{9}$ **b** $0.1\dot{8}$ **c** $0.2\dot{7}$, $0.3\dot{6}$ and $0.6\dot{3}$

11 **a** $0.\dot{2}85714\dot{}$ **b** $0.\dot{4}28571\dot{}$ **c** $\frac{4}{7} = 0.\dot{5}71428\dot{}$, $\frac{5}{7} = 0.\dot{7}14285\dot{}$ and $\frac{6}{7} = 0.\dot{8}57142\dot{}$

12 **a** $\frac{1}{5}$, $\frac{1}{8}$, $\frac{1}{10}$

b $\frac{1}{N}$ is a terminating decimal if the only prime factors of N are 2 or 5. Otherwise it is a recurring decimal.

2.4 Converting percentages, fractions and decimals

Exercise 2D

1 **a** $\frac{2}{25}$ **b** $\frac{1}{2}$ **c** $\frac{1}{4}$ **d** $\frac{7}{20}$ **e** $\frac{9}{10}$ **f** $\frac{7}{4}$ or $1\frac{3}{4}$

2 **a** 0.27 **b** 0.85 **c** 130% **d** 0.06 **e** 0.8 **f** 0.32

3 **a** $\frac{3}{25}$ **b** $\frac{2}{5}$ **c** $\frac{9}{20}$ **d** $\frac{17}{25}$ **e** $\frac{9}{4}$ or $2\frac{1}{4}$ **f** $\frac{5}{8}$

4 **a** 29% **b** 55% **c** 3% **d** 16% **e** 160% **f** 125%

5 **a** 28% **b** 30% **c** 95% **d** 8.5% **e** 162.5% **f** 450%

6 **a** 0.6 **b** 0.075 **c** 0.76 **d** 0.3125 **e** 1.45 **f** 1.125

7 **a** 63%, 83%, 39%, 62%, 77% **b** English

8 34%, 0.34, $\frac{17}{50}$; 85%, 0.85, $\frac{17}{20}$; 7.5%, 0.075, $\frac{3}{40}$; 45%, 0.45, $\frac{9}{20}$; 30%, 0.3, $\frac{3}{10}$; 67%, 0.67, $\frac{2}{3}$; 84%, 0.84, $\frac{21}{25}$; 45%, 0.45, $\frac{9}{20}$; 37.5%, 0.375, $\frac{3}{8}$

2.5 Calculating a percentage

Exercise 2E

1 **a** 0.88 **b** 0.3 **c** 0.25 **d** 0.08 **e** 1.15

2 **a** 78% **b** 40% **c** 75% **d** 5% **e** 110%

3 **a** $45 **b** $6.30 **c** 128.8 kg **d** 1.125 kg
e 1.08 h **f** 37.8 cm **g** $0.12 **h** 2.94 m
i $7.60 **j** 33.88 min **k** 136 kg **l** $162

4 $2410

5 **a** 86% **b** 215

6 8520

7 287

8 990

9 Mon: 816, Tue: 833, Wed: 850, Thu: 799, Fri: 748

10 **a** $3.25 **b** 2.21 kg **c** $562.80
d $6.51 **e** 42.93 m **f** $24

11 480 cm^3 nitrogen, 120 cm^3 oxygen

12 13

13 $270

14 More this year as it was 3% of a higher amount than last year.

2.6 Increasing or decreasing quantities by a percentage

Exercise 2F

1 **a** 1.1 **b** 1.03 **c** 1.2 **d** 1.07 **e** 1.12

2 **a** $62.40 **b** 12.96 kg **c** 472.5 g **d** 599.5 m
e $38.08 **f** $90 **g** 391 kg **h** 824.1 cm
i 253.5 g **j** $143.50 **k** 736 m **l** $30.24

3 $29 425

4 1 690 200

5 **a** Caretaker: $17 325, Driver: $18 165, Supervisor: $20 475, Manager: $26 565
b 5% of different amounts is not a fixed amount. The more pay to start with, the more the increase (5%) will be.

6 $411.95

7 193 800

8 575 g

9 918

10 60

11 TV: $ 287.88, microwave: $ 84.60, CD player: $ 135.13, stereo: $ 34.66

12 $10

13 **c** Both the same as 1.05 × 1.03 = 1.03 × 1.05

14 **a** Shop A, as 1.04 × 1.04 = 1.0816, so an 8.16% increase.

15 $540.96

Exercise 2G

1 **a** 0.92 **b** 0.85 **c** 0.75 **d** 0.91 **e** 0.88

2 **a** $9.40 **b** 23 kg **c** 212.4 g **d** 339.5 m
e $4.90 **f** 39.6 m **g** 731 m **h** 83.52 g
i 360 cm **j** 117 min **k** 81.7 kg **l** $37.70

3 $5525

4 **a** 70.4 kg **b** 83.6 kg **c** 95.04 kg

5 Mr Patel $176, Mrs Patel $297.50, Sandeep $341, Priyanka $562.50

6 448

7 705

8 **a** 66.5 km/h **b** 73.5 km/h

9 No, as the total is $101. She will save $20.20, which is less than the $25 it would cost to join the club.

10 10% off $50 is $45; 10% off $45 is $40.50; 20% off $50 is $40

11 $765

12 1.10 × 0.9 = 0.99 (99%)

13 Offer A gives 360 grams for $1.40, i.e. 0.388 cents per gram. Offer B gives 300 grams for $1.12, i.e. 0.373 cents per gram, so Offer B is the better offer.
Or Offer A is 360 for 1.40 = 2.6 grams per cent, offer B is 300 for 1.12 = 2.7 grams per cent, so offer B is better.

2.7 One quantity as a percentage of another

Exercise 2H

1 **a** 25% **b** 60.6% **c** 46.3% **d** 12.5%
e 41.7% **f** 60% **g** 20.8% **h** 10%
i 1.9% **j** 8.3% **k** 45.5% **l** 10.5%

2 32%

3 6.5%

4 33.7%

5 **a** 49.2% **b** 64.5% **c** 10.6%

6 17.9%

7 4.9%

8 90.5%

9 **a** Brit Com: 20.9%, USA: 26.5%, France: 10.3%, Other 42.3%
b Total 100%, all imports

10 Nadia had the greater percentage increase.
Nadia: (20 – 14) × 100 ÷ 14 = 42.9%.
Imran: (17 – 12) × 100 ÷ 12 = 41.7%

11 Yes, as 38 out of 46 is over 80% (82.6%)

12 Vase 20% loss, radio 25% profit, doll 175% profit, toy train 64% loss

2.8 Simple interest and compound interest

Exercise 2I

1 7420 dollars

2 3600 dollars

3 4 years

4 **a** $15 600 **b** $16 224

5 **a** $1272 **b** $1348.32 **c** $1429.22

6 **a** Amar 3200 $, Mona 3328 $ **b** Mona, 128 $

7 **a** $9528.13 **b** £1528.13

8 £3840

9 **a** Simple **b** 6.5%

10 **a** $13 800 **b** $15 870
c Student's own explanation

11 **a** 2652.25 and 5304.50 **b** £796.37

2.9 A formula for compound interest

Exercise 2J

1 $2249.73

2 $5681.15

3 **a** $5071.50 **b** $5591.33 **c** $6164.44

4 **a** $3589.07 **b** $4458.69

5 $4272.64

6 **a** $3941.57 **b** $441.57

7 8 years

8 The interest in the second five years will be more than the interest in the first five years. The missing number is $5000 \times 1.06^{10} = 8954.24$.

9 **a** $15 000 **b** $16 288.95

10 **a** $1268.24
b The interest over the year is $268.24.
This is $\frac{268.24}{1000} \times 100\% = 26.824\%$ of $1000.

2.10 Reverse percentage

Exercise 2K

1 **a** 800 g **b** 250 m **c** 60 cm
d $3075 **e** $200 **f** $400

2 80

3 T shirt: $8.40, Tights: $1.20, Shorts: $5.20, Sweater: $10.74, Trainers: $24.80, Boots: $32.40

4 $833.33

5 $300

6 240

7 537.63 dollars

8 4750 blue bottles

9 $2585

10 $1440

11 $2450

12 95 dollars

13 $140

14 $945

15 $1325

16 $1300

17 Lee has assumed that 291.2 is 100% instead of 112%.
He rounded his wrong answer to the correct answer of $260.

Answers to Chapter 3

3.1 Order of operations

Exercise 3A

1 **a** 11 **b** 6 **c** 10 **d** 12 **e** 11 **f** 13
g 11 **h** 12 **i** 12 **j** 4 **k** 13 **l** 3

2 **a** 16 **b** 2 **c** 10 **d** 10 **e** 6 **f** 18
g 6 **h** 15 **i** 9 **j** 12 **k** 3 **l** 8

3 **a** (4 + 1) **b** No brackets needed
c (2 + 1) **d** No brackets needed
e (4 + 4) **f** (16 – 4)
g No brackets needed **h** No brackets needed
i (20 – 10) **j** No brackets needed
k (5 + 5) **l** (4 + 2)
m (15 – 5) **n** (7 – 2)
o (3 + 3) **p** No brackets needed
q No brackets needed **r** (8 – 2)

4 No, correct answer is 5 + 42 = 47

5 **a** 2 × 3 + 5 = 11 **b** 2 × (3 + 5) = 16
c 2 + 3 × 5 = 17 **d** 5 – (3 – 2) = 4
e 5 x 3 – 2 = 13 **f** 5 × 3 × 2 = 30

6 4 + 5 × 3 = 19
(4 + 5) × 3 = 27. So 4 + 5 × 3 is smaller

7 (5 – 2) × 6 = 18

8 8 ÷ (5 – 3) = 4

3.2 Choosing the correct operation

Exercise 3B

1 **a** 6000
b 5 cans cost $1.95, so 6 cans cost $1.95. 32 = (5 × 6) + 2. Cost is $10.53.

2 **a** 288 **b** 16

3 **a** 38
b Coach price for adults = $8, coach price for juniors = $4, money for coaches raised by tickets = $12 400, cost of coaches = $12 160, profit = $240

4 (39 × 20) + (90 × 30) = 1050 = $10.50

5 (18.81…) Kirsty can buy 18 models.

6 (7.58 …) Michaela must work for 8 weeks.

7 $8.40 per year, 70 cents per copy

8 $450

9 15

10 Gustav pays 2296.25 – 1840 = $456.25

3.3 Finding a fraction of a quantity

Exercise 3C

1 **a** 18 **b** 10 **c** 18 **d** 28

2 **a** $1800 **b** 128 g **c** 160 kg
d $116 **e** 65 litres **f** 90 min

3 **a** $\frac{5}{8}$ of 40 = 25 **b** $\frac{3}{4}$ of 280 = 210
c $\frac{4}{5}$ of 70 = 56 **d** $\frac{5}{6}$ of 72 = 60

4 $6080

5 $31 500

6 52 kg

7 **a** 856 **b** 187 675

8 **a** $50 **b** $550

9 **a** $120 **b** $240

10 Lion Autos

11 Offer B

3.4 Adding and subtracting fractions

Exercise 3D

1 **a** $\frac{5}{7}$ **b** $\frac{7}{9}$ **c** $\frac{4}{5}$ **d** $\frac{6}{7}$

2 **a** $\frac{3}{7}$ **b** $\frac{1}{9}$ **c** $\frac{4}{11}$ **d** $\frac{7}{13}$

3 **a** $\frac{6}{8}=\frac{3}{4}$ **b** $\frac{4}{10}=\frac{2}{5}$ **c** $\frac{6}{9}=\frac{2}{3}$ **d** $\frac{2}{4}=\frac{1}{2}$

4 **a** $\frac{4}{8}=\frac{1}{2}$ **b** $\frac{4}{10}=\frac{2}{5}$ **c** $\frac{4}{6}=\frac{2}{3}$ **d** $\frac{8}{10}=\frac{4}{5}$

5 **a** $\frac{12}{10}=\frac{6}{5}=1\frac{1}{5}$ **b** $\frac{9}{8}=1\frac{1}{8}$ **c** $\frac{9}{8}=1\frac{1}{8}$
d $\frac{13}{8}=1\frac{5}{8}$ **e** $\frac{11}{8}=1\frac{3}{8}$ **f** $\frac{7}{6}=1\frac{1}{6}$
g $\frac{9}{6}=\frac{3}{2}=1\frac{1}{2}$ **h** $\frac{5}{4}=1\frac{1}{4}$

6 **a** $\frac{10}{8}=\frac{5}{4}=1\frac{1}{4}$ **b** $\frac{6}{4}=\frac{3}{2}=1\frac{1}{2}$
c $\frac{5}{5}=1$ **d** $\frac{16}{10}=\frac{8}{5}=1\frac{3}{5}$

7 **a** $\frac{5}{8}$ **b** $\frac{5}{10}=\frac{1}{2}$ **c** $\frac{1}{4}$ **d** $\frac{3}{8}$
e $\frac{1}{4}$ **f** $\frac{3}{8}$ **g** $\frac{4}{10}=\frac{2}{5}$ **h** $\frac{5}{16}$

Exercise 3E

1 **a** $\frac{8}{15}$ **b** $\frac{7}{12}$ **c** $\frac{3}{10}$ **d** $\frac{11}{12}$ **e** $\frac{7}{8}$ **f** $\frac{1}{2}$
g $\frac{1}{6}$ **h** $\frac{1}{20}$ **i** $\frac{1}{10}$ **j** $\frac{1}{8}$ **k** $\frac{1}{12}$ **l** $\frac{1}{3}$
m $\frac{1}{6}$ **n** $\frac{7}{9}$ **o** $\frac{5}{8}$ **p** $\frac{3}{8}$ **q** $\frac{1}{15}$ **r** $1\frac{13}{24}$
s $\frac{59}{80}$ **t** $\frac{22}{63}$ **u** $\frac{37}{54}$

2 **a** $3\frac{5}{14}$ **b** $10\frac{3}{5}$ **c** $2\frac{1}{6}$ **d** $3\frac{31}{45}$
e $4\frac{47}{60}$ **f** $\frac{41}{72}$ **g** $\frac{29}{48}$ **h** $1\frac{43}{48}$
i $1\frac{109}{120}$ **j** $1\frac{23}{30}$ **k** $1\frac{31}{84}$

3 $\frac{1}{20}$

4 **a** $\frac{1}{6}$ **b** 30, must be divisible by 2 and 3

3.5 Multiplying and dividing fractions

Exercise 3F

1 **a** $\frac{1}{6}$ **b** $\frac{1}{10}$ **c** $\frac{3}{8}$ **d** $\frac{3}{14}$ **e** $\frac{8}{15}$
f $\frac{1}{5}$ **g** $\frac{2}{7}$ **h** $\frac{3}{10}$ **i** $\frac{1}{2}$ **j** $\frac{2}{5}$

2 **a** $\frac{3}{32}$ **b** $\frac{3}{8}$ **c** $\frac{7}{20}$
d $\frac{16}{45}$ **e** $\frac{3}{5}$ **f** $\frac{5}{8}$

3 $\frac{1}{12}$

4 $\frac{3}{8}$

5 **a** $\frac{5}{12}$ **b** $2\frac{1}{12}$
c $6\frac{1}{4}$ **d** $2\frac{11}{12}$
e $3\frac{9}{10}$ **f** $3\frac{1}{3}$
g $12\frac{1}{2}$ **h** 30

6 $\frac{2}{5}$ of $6\frac{1}{2} = 2\frac{3}{5}$

Exercise 3G

1 **a** $\frac{3}{4}$ **b** $1\frac{2}{5}$ **c** $1\frac{1}{15}$ **d** $1\frac{1}{14}$ **e** 4
f 4 **g** 5 **h** $1\frac{5}{7}$ **i** $\frac{4}{9}$ **j** $1\frac{3}{5}$

2 18

3 40

4 15

5 16

6 **a** $2\frac{2}{15}$ **b** 38 **c** $1\frac{7}{8}$ **d** $\frac{9}{32}$ **e** $\frac{1}{16}$ **f** $\frac{256}{625}$

7 **a** $1\frac{1}{3}$ **b** $\frac{3}{4}$

Answers to Chapter 4

4.1 Introduction to negative numbers

Exercise 4A

1 **a** 0 °C **b** 5 °C **c** –2 °C **d** –5 °C **e** –1 °C

2 **a** 11 degrees Celsius **b** 9 degrees Celsius

3 8 degrees Celsius

4 38 degrees Celsius

5 **a** 2 degrees Celsius between Helsinki and Moscow
b 34 degrees Celcius between Dubai and Helsinki

4.2 Everyday use of negative numbers

Exercise 4B

1 –$5

2 –200 m

3 above

4 –5 h

5 –2 °C

6 – 70 km

7 +5 minutes

8 –5 km/h

9 –2

10 **a** –11 °C **b** 6 degrees Celsius

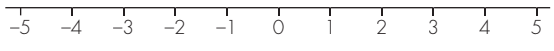
11 1.54 am

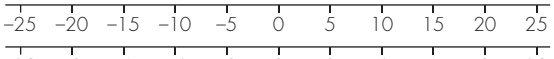
4.3 The number line

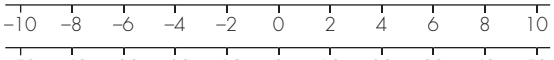
Exercise 4C

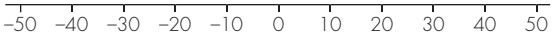
1 **a** < **b** > **c** < **d** < **e** > **f** <
g < **h** > **i** > **j** < **k** < **l** >

2 **a** < **b** < **c** < **d** > **e** < **f** <

3 **a**


c


d


e


4 6 °C –2 °C –4 °C 2 °C

5 **a** 1 or 0 or –1 or –2 are the possible answers **b** No solution
c Any integer larger than 2. That is 3 or 4 or 5 or …
d Any integer smaller than –3. That is –4 or –5 or –6 or …

4.4 Adding and subtracting with negative numbers

Exercise 4D

1 **a** –2 **b** –3 **c** –2 **d** –13 **e** –2 **f** –3
g 3 **h** 3 **i** –1 **j** –1 **k** 2 **l** –3
m –4.5 **n** –6.5 **o** –6.8 **p** –1.6 **q** –5 **r** –4
s 4 **t** –11 **u** –5.6 **v** –3.7 **w** –5 **x** –5.3

2 **a** 7 degrees Celsius **b** –6 °C

3 **a** 2 – 8
b 2 + 5 – 8 or 2 + 4 – 7 or 8 – 4 – 5 or 8 – 2 – 7 or 5 – 4 – 2
c 2 – 5 – 7 – 8
d 2 + 5 – 4 – 7 – 8

4 250 metres

Exercise 4E

1 **a** –8 **b** –10 **c** –11 **d** –3 **e** 2 **f** –5
g 1 **h** 4 **i** 7 **j** –8 **k** –16.4 **l** –112.1
m 11 **n** 6 **o** 8.5 **p** 9 **q** –112.1 **r** 6
s –9 **t** –5

2 **a** 10 degrees Celsius
b 7 degrees Celsius
c 29 degrees Celsius

3 **a** 2 **b** –3 **c** –5 **d** –7 **e** –10 **f** –20

4 **a** 2 **b** 4 **c** –1 **d** –5 **e** –11 **f** 8

5 **a** 13 **b** 2 **c** 5 **d** 3.5 **e** 11.2 **f** –2

6 **a** –10 **b** –5 **c** –2 **d** 4 **e** 7 **f** –7.5

7 **a** +6 + +5 = 11 **b** +6 + –9 = –3
c +6 – –9 = 15 **d** +6 – +5 = 1

8 It may not come on as the thermometer inaccuracy might be between 0° and 2° or 2° and 4°

9 9 and –4

4.5 Multiplying and dividing with negative numbers

Exercise 4F

1 **a** –15 **b** –14 **c** –24 **d** 6 **e** 14 **f** 2
g –2 **h** –8 **i** –4 **j** 3 **k** –24 **l** –10
m –18 **n** 16 **o** 36 **p** –4 **q** –12 **r** –4
s 7 **t** 25 **u** 18

2 **a** –9 **b** 16 **c** –3 **d** –32 **e** 18 **f** 18
g 6 **h** –4 **i** 20 **j** 16 **k** 8 **l** –48
m 13 **n** –13 **o** –8 **p** 0 **q** 16 **r** –42

3 **a** –2 **b** 30 **c** 15 **d** –27 **e** –7

4 **a** 4 **b** –9 **c** –3 **d** 6 **e** –4

5 **a** –9 **b** 3 **c** 1

6 **a** 16 **b** –2 **c** –12

7 **a** 24 **b** 6 **c** –4 **d** –2

8 For example: 1 × (–12), –1 × 12, 2 × (–6), 6 × (–2), 3 × (–4), 4 × (–3)

9 For example: 4 ÷ (–1), 8 ÷ (–2), 12 ÷ (–3), 16 ÷ (–4), 20 ÷ (–5), 24 ÷ (–6)

10 –5 × 4, 3 × –6, –20 ÷ 2, –16 ÷ –4

11 **a** 4 **b** 25 **c** 12 **d** 1

12

×	–2	3	–4
–5	10	–15	20
2	–4	6	–8
–6	12	–18	24

Answers to Chapter 5

5.1 Squares and square roots

Exercise 5A

1 **a** 49 **b** 100 **c** 1.44 **d** 6.25 **e** 256 **f** 400
g 9.61 **h** 20.25 **i** 9 **j** 64 **k** 0.25 **l** 0.25

2 **a** 3 and –3 **b** 10 and –10 **c** 4 and –4
d 12 and –12 **e** 2 and –2 **f** 20 and –20
g 13 and –13 **h** 9 and –9

3 **a** 5 **b** 6 **c** 10 **d** 7 **e** 8
f 11 **g** 13 **h** 15 **i** 20 **j** 14

4 **a** 81 **b** 40 **c** 100 **d** 14 **e** 36
f 15 **g** 49 **h** 12 **i** 25 **j** 21

5 **a** 24 **b** 31 **c** 45 **d** 40 **e** 67
f 101 **g** 3.6 **h** 6.5 **i** 13.9 **j** 22.2

6 $\sqrt{50}$, 3^2, $\sqrt{90}$, 4^2

7 **a** 6^2 is 36 and 7^2 is 49; 40 is between 36 and 49
b 6.3245553……

8 4 and 5

9 **a** 8 and 9 **b** 9 and 10
c 11 and 13 **d** 14 and 15

10 $\sqrt{324} = 18$

11 15

5.2 Cubes and cube roots

Exercise 5B

1 **a** 8 **b** 27 **c** 512 **d** 1000
e 1.331 **f** 15.625 **g** –27 **h** –125
i 8000 **j** 68.921 **k** –68.921

2 **a** 2 **b** 5 **c** 9 **d** 1 **e** 3
f –3 **g** 10 **h** 1.5 **i** 4.5 **j** 0.5

3 **a** 2 and 3 **b** 1 and 2
c 4 and 5 **d** 3 and 4

4 2^3 because it equals 8, the rest equal 9

5 One possible answer is $8^2 = 4^3$

6 $\sqrt[3]{2000}$, $\sqrt{225}$, 2.5^3, 4^2

7

Number	Square	Cube
10	100	1000
5	25	125
4	16	64
11	121	1331
9	81	729

8 0.8^3, 0.8^2, $\sqrt{0.8}$, $\sqrt[3]{0.8}$.

5.3 More powers and roots

Exercise 5C

1 **a** 243 **b** 2401 **c** 1 000 000 **d** 256

2 **a** 0 **b** 118 **c** 513

3 **a** 2592 **b** 227 **c** or 0.3789 to 4 d.p.

4 LHS = 31; RHS = 32 – 1 = 31

5 **a** 5 **b** 11 **c** 3 **d** 20

6 **a** 20 000 **b** or 1.125

7 $2^3 \times \sqrt[3]{8} = 16$ $3^2 \times \sqrt[10]{1024} = 18$ $\sqrt[3]{64} \times \sqrt[3]{125} = 20$
$\sqrt[4]{14641} \times \sqrt[6]{64} = 22$

8 **a** 32 **b** 12.5

9 **a** $8\frac{1}{3}$ **b** $\frac{7}{8}$

10 **a** 3 **b** 8 **c** 5

11 **a** 3 **b** 4 **c** 8

12 **a** 15 625 **b** 1 953 125

13 **a** $x = 4$ and $y = 2$ is one possible pair.

b $x = 8$ and $y = 4$ is a second possible pair; $x = 12$ and $y = 6$ is a third possible pair

14 **a** 1024 **b** 1 048 576 **c** 32 **d** 4

e 2

5.4 Exponential growth and decay

Exercise 5D

1 **a i** 10 million **ii** 20 million **iii** 40 million

b i 15 million **ii** 45 million **iii** 135 million

2 **a** 6000 **b** 9000 **c** 13 500 **d** 20 250

3 **a** 6000 **b** 1500 **c** 375

4 **a** 4800 **b** 768 **c** 123

5 21.8, 23.8, 25.9

6 18.2, 16.6, 15.1

7 **a** \$6312 **b** 6 years

8 **a** \$100 000 **b** \$195 313

9 The correct value is $150\,000 \times 1.2^5 = 373\,248$

10 **a** \$20 **b** \$20 480

11 **a** 1185 **b** 351

12 272 million

13 **a** \$18 **b** \$32 **c** \$340 **d** \$11568

Answers to Chapter 6

6.1 Inequalities

Exercise 6A

1 **a** $>$ **b** $<$ **c** $<$ **d** $=$

e $=$ **f** $>$ **g** $>$ **h** $<$

2 $\frac{1}{3} < \frac{1}{2} < \frac{3}{5}$

3 **a** 4,5,6 **b** 1,2 **c** 6 **d** 1,2,3,4,5

e 2,3,4 **f** 4,5 **g** 1,2,3 **h** 6

4 **a** underweight **b** overweight

c normal **d** normal

5 20, 22, 26, 28

6 **a** 49 **b** 45

c 3,6,9 **d** 16,17,18,19,20

7 **a** true **b** false **c** true

d true **e** false **f** true

8 **a** 6,7,8 **b** 26, 27, 28 **c** –7, –6, –5, –4

d –2, –1, 0, 1 **e** there are none **f** 33

9 **a** $N \geqslant 8$ or $N \leqslant -8$ **b** $N \geqslant 4$ **c** $N < -4$ **d** no solution

6.2 Sets and Venn diagrams

Exercise 6B

1 **a** The elements can be listed in any order.

i {2, 3, 12, 4, 10, 11} **ii** {4, 5, 11, 10}

iii {4, 10, 11}

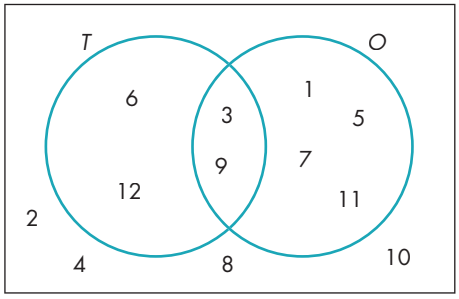
b i 6 **ii** 7 **iii** 12

c The first 12 natural numbers.

2 **a i** 7 **ii** 5 **iii** 2 **iv** 10

b a, p, r, l or n **c** g or d

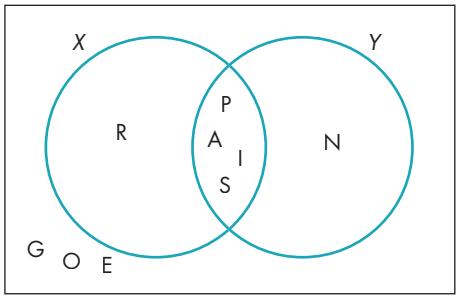
3



4 **a** 100 is in A but not in B **b** 6

c multiples of 6.

5



6 **a** 15 **b** 21 **c** 35

7 **a** 8 **b** 10 **c** {1, 2, 4} **d** {2, 7}

8 **a** {1, 2, 3, 4, 5, 6, 7, 8, 10, 13} **b** {1, 2, 4, 6, 7, 8, 10, 13}

c {2, 4, 6} **d** {4, 10}

9 **a** {8, 11, 12, 14, 15} **b** {6, 9, 14, 15} **c** {14, 15}

d {8, 11, 12} **e** {6, 9}

10 **a** True **b** False **c** True

d True **e** False **f** False

11 **a i** $X \cap Y'$ is 3 **ii** $(X \cap Y)'$ is 2 **iii** $X \cup Y'$ is 4

iv $(X \cup Y)'$ is 1 **v** $X' \cap Y'$ is 1 **vi** $X' \cup Y'$ is 2

b $(X \cap Y)' = X' \cup Y'$ because they have the same diagram.

Also $(X \cup Y)' = X' \cap Y'$

6.3 More about Venn diagrams

Exercise 6C

1 **a** **i** $A = \{4, 22, 2, 20, 12, 10, 28, 30\}$

ii $B = \{2, 20, 12, 6, 26, 16, 18\}$

iii $A \cap B = \{10, 12, 28, 30\}$

iv $A \cap C = \{2, 20, 12\}$ $A \cap B \cap C = \{12\}$

b **i** 7 **ii** 10 **iii** 12 **iv** 13 **c** The even numbers up to 30.

2 **a** **i** $X = \{1, 2, 5, 10\}$ **ii** $Y = \{1, 2, 3, 4, 6, 8, 12, 24\}$

b **i** factors of 10 **ii** factors of 24

c **i** {1, 2, 3, 6} **ii** {1, 2} **iii** [1, 2, 3]

d **i** 8 **ii** 10 **iii** 12

3 **a** **i** {10} **ii** {7, 15} **iii** none

b **i** 6 **ii** 9 **iii** 7

iv 14 **v** 12

4 **a** **i** {4, 8, 12} **ii** {2, 4, 6, 8, 10, 12, 14}

iii {10} **iv** {4, 8, 12}

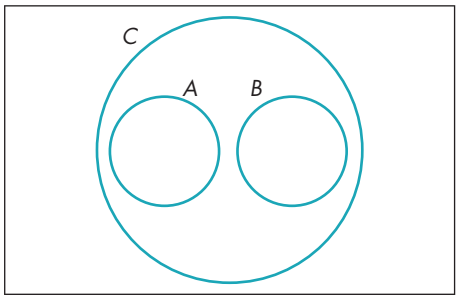
b **i** 1 **ii** 0

5 **a** **i** {a, b, c, d, e, f} **ii** {d, e, f}

iii {b, c, d, e, f, g} **iv** {b, d, e}

b 3

6 **a**



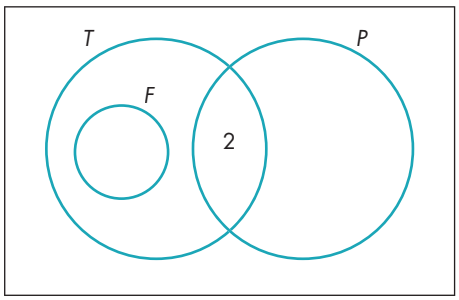
b C

7 $n(A) + n(B)$ counts the elements in the intersection twice but $n(A \cup B)$ only counts them once. This means that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

If $n(A \cup B) = n(A) + n(B)$ there are no elements in the intersection. Hence $A \cap B = \varnothing$

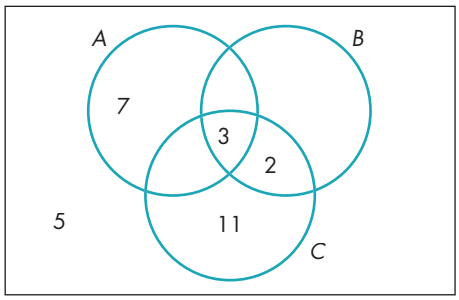
8 **a** $x = 2$ because 2 is the only even prime number.

b



c T = {even numbers} so T' = {odd numbers}

9

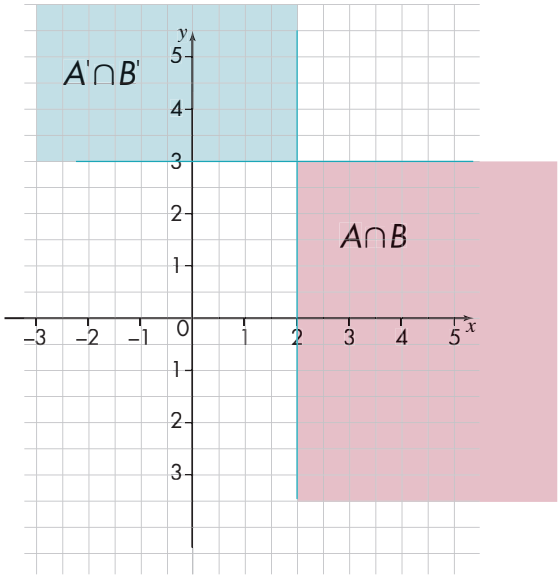


10 **a** $C \subseteq B$ is true **b** $A \cap C$ is true

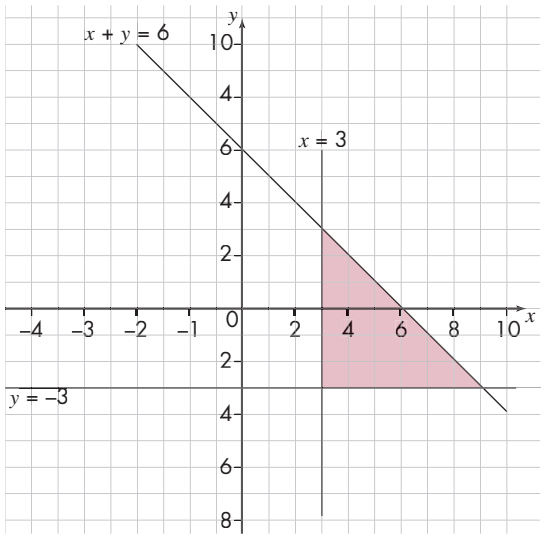
c It is false; $C \cup B = B$ **d** It is false; $B \cap C = C$

e It is false; $A' \cap C = C$ **f** It is false; $C' \cup A' = \xi$

11 **a** and **b**

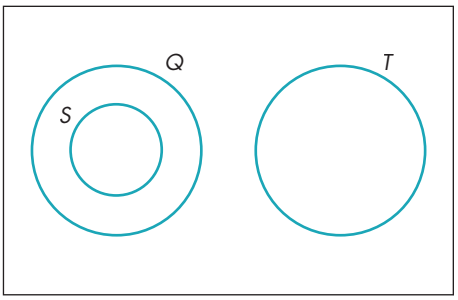


12 **a**



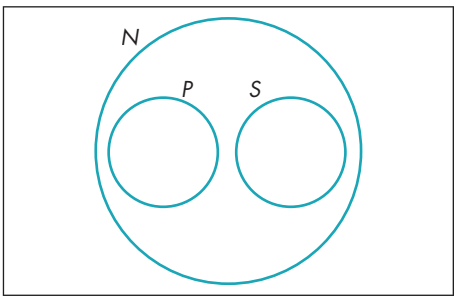
b Any coordinates of the form (0, c) where $-3 \leqslant c \leqslant 6$

13 a



b $R \cap T$ = {equilateral triangles}

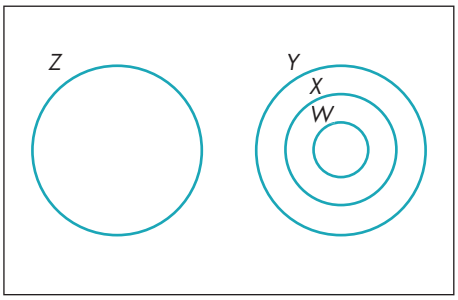
14 a



b No prime number is a square number and so $S \cap P = \varnothing$

c The smallest natural number that is not prime or square is 6.

15 A Venn diagram shows the sets



a $W \cap Y = W$ **b** $W \cup Y = Y$ **c** $X' \cap Z = Z$ **d** $Z' \cup Y = Z'$

Answers to Chapter 7

7.1 Ratio

Exercise 7A

1 **a** 1 : 3 **b** 1 : 4 **c** 2 : 3 **d** 2 : 1
e 2 : 5 **f** 2 : 5 **g** 5 : 8 **h** 5 : 1

2 **a** 8 : 1 **b** 12 : 1 **c** 5 : 6 **d** 1 : 24
e 48 : 1 **f** 5 : 2 **g** 3 : 8 **h** 1 : 5

3 $\frac{7}{10}$

4 $\frac{10}{25} = \frac{2}{5}$

5 **a** $\frac{2}{5}$ **b** $\frac{3}{5}$

6 **a** $\frac{7}{10}$ **b** $\frac{3}{10}$

7 **a** $\frac{1}{2}$ **b** $\frac{7}{20}$ **c** $\frac{3}{20}$

8 3 : 1

Exercise 7B

1 **a** 160 g, 240 g **b** 80 kg, 200 kg
c 150, 350 **d** 950 m, 50 m
e 175 min, 125 min **f** \$20, \$30, \$50
g \$36, \$60, \$144 **h** 50 g, 250 g, 300 g
i \$1.40, \$2, \$1.60 **j** 120 kg, 72 kg, 8 kg

2 **a** 175 **b** 30%

3 **a** 28 **b** 42

4 21

5 Joshua \$2500, Aicha \$3500, Mariam \$4000

6 **a** 1 : 400 000 **b** 1 : 125 000 **c** 1 : 250 000
d 1 : 25 000 **e** 1 : 20 000 **f** 1 : 40 000
g 1 : 62 500 **h** 1 : 10 000 **i** 1 : 60 000

7 **a** 1 : 1 000 000 **b** 47 km **c** 8 mm

8 **a** 1 : 250 000 **b** 2 km **c** 4.8 cm

9 **a** 1 : 20 000 **b** 0.54 km **c** 40 cm

10 **a** 1 : 1.6 **b** 1 : 3.25 **c** 1 : 1.125
d 1 : 1.44 **e** 1 : 5.4 **f** 1 : 1.5
g 1 : 4.8 **h** 1 : 42 **i** 1 : 1.25

11 **c** 1 : 250 000 At this scale 134 km is 53.6 cm which is a sensible size. The others are too small (5.36 mm or 5.36 cm) or too large (5.36 m).

Exercise 7C

1 **a** 3 : 2 **b** 32 **c** 80

2 1000 g

3 10 125

4 **a** 14 min **b** 75 min

5 **a** 11 pages **b** 32%

6 Ren \$2040, Shota \$2720

7 **a** lemonade 20 litres, ginger 0.5 litres
b This one, one-thirteenth is greater than one-fiftieth.

7.2 Speed

Exercise 7D

1 18 km/hour

2 440 kilometres

3 52.5 km/hour

4 11.50 am

5 500 s

6 **a** 75 km/hour **b** 6.5 hours **c** 175 km **d** 240 km
e 64 km/h **f** 325 km **g** 4.3 hours (4 h 18 min)

7 **a** 7.75 h **b** 85.2 km/hour
8 **a** 2.25 h **b** 157.5 km
9 **a** 1.25 h **b** 1 h 15 min
10 **a** 48 km/hour **b** 6 h 40 min
11 **a** 120 km **b** 48 km/h
12 **a** 30 min **b** 12 km/h
13 **a** 10 m/s **b** 3.3 m/s **c** 16.7 m/s **d** 41.7 m/s **e** 20.8 m/s
14 **a** 90 km/h **b** 43.2 km/h **c** 14.4 km/h **d** 108 km/h **e** 1.8 km/h
15 **a** 64.8 km/h **b** 28 s **c** 8.07 (37 min journey)
16 **a** 6.7 m/s **b** 66 km **c** 5 minutes **d** 133.3 metres
17 6.6 minutes

7.3 Rates

Exercise 7E

1 **a** 3.5 cm **b** 20 days
2 **a** 15 litres **b** 20 seconds
3 **a** 10.59 g/cm^3 **b** 50 cm^3
4 1600 days (4.38 years)
5 **a** 0.5 mm/year **b** 12.5 mm **c** 40 years
6 **a** 4.44 cm^3 to 3 d.p. **b** 360 g
c No. The density is 8 g/cm^3, not 9 g/cm^3
7 **a** 62.5 Pa or N/m^2 **b** 250 Pa or N/m^2
c 187.5 Pa or N/m^2
8 **a** 31.8 litres **b** 5.3 litres **c** 1.06 **d** 0.106 **e** 943 km
9 **a** 14.5 kg **b** 2900 kg **c** About 8 kg/day
10 **a** Indonesia 141 people/km^2 Spain 93 people/km^2 **b** Yes
11 196 million
12 **a** $579.60 **b** 33 hours
c Marina earns $14.75 per hour. This is more than Asif's $12.60.
13 **a** $34 140 **b** 18 months

7.4 Proportion

Exercise 7F

1 60 g
2 $5.22
3 45
4 $6.72
5 **a** $312.50 **b** 8
6 **a** 56 litres **b** 350 km
7 **a** 300 kg **b** 9 weeks
8 40 s
9 **a** **i** 100 g margarine, 200 g sugar, 250 g flour, 150 g ground rice
ii 150 g margarine, 300 g sugar, 375 g flour, 225 g ground rice
iii 250 g margarine, 500 g sugar, 625 g flour, 375 g ground rice
b 24
10 Pieter's shop, as I can buy 24. At Paulo's shop I can only buy 20.

Answers to Chapter 8

8.1 Rounding whole numbers

Exercise 8A

1 **a** 20 **b** 60 **c** 80 **d** 50 **e** 100
f 20 **g** 90 **h** 70 **i** 10 **j** 30
2 **a** 200 **b** 600 **c** 800 **d** 500 **e** 1000
f 100 **g** 600 **h** 400 **i** 1000 **j** 1100
3 **a** 2000 **b** 6000 **c** 8000 **d** 5000
e 10 000 **f** 1000 **g** 6000 **h** 3000
i 9000 **j** 2000
4 **a** True **b** False **c** True **d** True **e** True **f** False
5 **a** Highest Germany, lowest Italy
b 36 000, 43 000 , 25 000, 29 000
c 25 499 and 24 500
6 **a** 375 **b** 25 (350 to 374 inclusive)
7 A number between 75 and 84 inclusive added to a number between 45 and 54 inclusive with a total not equal to 130, for example 79 + 49 = 128

8.2 Rounding decimals

Exercise 8B

1 **a** 4.8 **b** 3.8 **c** 2.2 **d** 8.3 **e** 3.7
f 46.9 **g** 23.9 **h** 9.5 **i** 11.1 **j** 33.5
2 **a** 5.78 **b** 2.36 **c** 0.98 **d** 33.09 **e** 6.01
f 23.57 **g** 91.79 **h** 8.00 **i** 2.31 **j** 23.92
3 **a** 4.6 **b** 0.08 **c** 45.716 **d** 94.85 **e** 602.1
f 671.76 **g** 7.1 **h** 6.904 **i** 13.78 **j** 0.1
4 **a** 8 **b** 3 **c** 8 **d** 6 **e** 4
f 7 **g** 2 **h** 47 **i** 23 **j** 96
5 3 + 9 + 6 + 4 = 22 dollars
6 3, 3.46, 3.5
7 4.7275 or 4.7282

8.3 Significant figures

Exercise 8C

1 **a** 50 000 **b** 60 000 **c** 30 000 **d** 90 000
e 90 000 **f** 0.5 **g** 0.3 **h** 0.006
i 0.05 **j** 0.0009 **k** 10 **l** 90
m 90 **n** 200 **o** 1000

2 **a** 56 000 **b** 27 000 **c** 80 000 **d** 31 000
e 14 000 **f** 1.7 **g** 4.1 **h** 2.7
i 8.0 **j** 42 **k** 0.80 **l** 0.46
m 0.066 **n** 1.0 **o** 0.0098

3 **a** 60 000 **b** 5300 **c** 89.7 **d** 110
e 9 **f** 1.1 **g** 0.3 **h** 0.7
i 0.4 **j** 0.8 **k** 0.2 **l** 0.7

4 **a** 65, 74 **b** 95, 149 **c** 950, 1499

5 Satora 750, 849, Nimral 1150, 1249, Korput 164 500, 165 499

6 One, because there could be 450 then 449.

7 Vashti has rounded to 2 significant figures or nearest 10 000.

8.4 Estimating answers to calculations

Exercise 8D

1 **a** 20 **b** 24 **c** 900 **d** 400 **e** 0.18 or 0.2

2 **a** 24 m **b** 48 m^2

3 **a** **i** 9 **ii** 27
b More in both cases

4 **a** 6 **b** 76 or 80 **c** 30 **d** 0.7 **e** 190 or 200

5 4 m/s

6 $120

7 90 m^3

8 **a** 20 **b** 2 **c** 80 **d** 4 **e** 0.5

9 150 yen (Accept 130, 135, 140)

10 20 s

11 315 ÷ 72 × 480 ≈ 300 ÷ 70 × 500 ≈ 4 × 500 = 2000 g or 2 kg

8.5 Upper and lower bounds

Exercise 8E

1 **a** 6.5 and 7.5 **b** 115 and 125
c 3350 and 3450 **d** 49.5 and 50.5
e 5.50 and 6.50 **f** 16.75 and 16.85
g 15.5 and 16.5 **h** 14 450 and 14 550
i 54.5 and 55.5 **j** 52.5 and 57.5

2 **a** 5.5 ⩽ length in cm < 6.5
b 16.5 ⩽ mass in kg < 17.5
c 31.5 ⩽ time in minutes < 32.5
d 237.5 ⩽ distance in km < 238.5
e 7.25 ⩽ distance in m < 7.35
f 25.75 ⩽ mass in kg < 25.85
g 3.35 ⩽ time in hours < 3.45
h 86.5 ⩽ mass in g < 87.5
i 4.225 ⩽ distance in mm < 4.235
j 2.185 ⩽ mass in kg < 2.195
k 12.665 ⩽ time in minutes < 12.675
l 24.5 ⩽ distance in metres < 25.5
m 35 ⩽ length in cm < 45
n 595 ⩽ mass in g < 605
o 25 ⩽ time in minutes < 35
p 995 ⩽ distance in metres < 1050
q 3.95 ⩽ distance in metres < 4.05
r 7.035 ⩽ mass in kg < 7.045
s 11.95 ⩽ time in seconds < 12.05
t 6.995 ⩽ distance in metres < 7.005

3 **a** 7.5, 8.5 **b** 25.5, 26.5
c 24.5, 25.5 **d** 84.5, 85.5
e 2.395, 2.405 **f** 0.15, 0.25
g 0.055, 0.065 **h** 250 g, 350 g
i 0.65, 0.75 **j** 365.5, 366.5
k 165, 175 **l** 205, 215

4 C: The chain and distance are both any value between 29.5 and 30.5 metres, so there is no way of knowing if the chain is longer or shorter than the distance.

5 2 kg 450 grams

6 **a** <65.5 g **b** 64.5 g
c <2620 g **d** 2580 g

8.6 Upper and lower bounds for calculations

Exercise 8F

1 65 kg and 75 kg

2 **a** 12.5 kg **b** 20

3 9 kg 53.5 – 44.5

4 **a** 26 cm ⩽ perimeter < 30 cm
b 25.6 cm ⩽ perimeter < 26.0 cm
c 50.5 cm ⩽ perimeter < 52.7 cm

5 **a** 38.25 cm^2 ⩽ area < 52.25 cm^2
b 37.1575 cm^2 ⩽ area < 38.4475 cm^2
c 135.625 cm^2 ⩽ area < 145.225 cm^2

6 79.75 m^2 ⩽ area < 100.75 m^2

7 216.125 cm^3 ⩽ volume < 354.375 cm^3

8 12.5 metres

9 Yes, because they could be walking at 4.5 km/h and 2.5 km/h meaning that they would cover 4.5 km + 2.5 km = 7 km in 1 hour

10 20.9 m ⩽ length < 22.9 m (3 sf)

11 **a** 14.65 s ⩽ time < 14.75 s
b 99.5 m ⩽ length < 100.5 m
c 6.86 m/s (3 sf)

12 14 s ⩽ time < 30 s

13 337.75 and 334.21

14 177.3 and 169.4

Answers to Chapter 9

9.1 Standard form

Exercise 9A

1 **a** 250 **b** 34.5 **c** 0.00467 **d** 34.6
e 0.020789 **f** 5678 **g** 246 **h** 7600
i 897 000 **j** 0.008 65 **k** 60 000 000 **l** 0.000567

2 **a** 2.5×10^{2} **b** 3.45×10^{-1} **c** 4.67×10^{4}
d 3.4×10^{9} **e** 2.078×10^{10} **f** 5.678×10^{-4}
g 2.46×10^{3} **h** 7.6×10^{-2} **i** 7.6×10^{-4}
j 9.99×10^{-1} **k** 2.3456×10^{2} **l** 9.87654×10^{1}
m 6×10^{-4} **n** 5.67×10^{-3} **o** 5.60045×10^{1}

3 2.7797×10^{4}

4 $3.211\,97 \times 10^{5}$, $4.491\,863 \times 10^{6}$

5 1.298×10^{7}, 2.997×10^{9}, 9.3×10^{4}

6 100

7 61.8 kilometres.

8 7.78×10^{8}; 5.8×10^{7}; 5.92×10^{9}

9.2 Calculating with standard form

Exercise 9B

1 **a** 5.67×10^{3} **b** 6×10^{2} **c** 3.46×10^{-1}
d 7×10^{-4} **e** 5.6×10^{2} **f** 6×10^{5}
g 7×10^{3} **h** 1.6 **i** 2.3×10^{7}

2 **a** 1.08×10^{8} **b** 4.8×10^{6} **c** 1.2×10^{9}
d 1.08 **e** 6.4×10^{2} **f** 1.2×10^{1}
g 2.88 **h** 2.5×10^{7} **i** 8×10^{-6}

3 **a** 2.7×10 **b** 1.6×10^{-2} **c** 2×10^{-1}
d 4×10^{-8} **e** 2×10^{5} **f** 6×10^{-2}

4 2×10^{13}, 1×10^{-10}, mass = 2×10^{3} g

5 **a** (2^{63}), 9.2×10^{18} grains
b $2^{64} - 1 = 1.8 \times 10^{19}$

6 **a** 1.0×10^{8} sq km
b 31%

7 3.80×10^{7} sq km

8 5×10^{4}

9 2.3×10^{5}

10 4.55×10^{8} kg or 455 070 tonnes

11 **a** 100 000 000 (100 million) **b** 1.4%

12 **a** 2.048×10^{6} **b** 4.816×10^{6}

13 9.41×10^{4}

14 Any value from 1.00000001×10^{8} to 1×10^{9} (excluding 1×10^{9}), i.e. any value of the form $a \times 10^{8}$ where $1 < a < 10$

15 **a** India **b** India and Jamaica
c 2.2×10^{7} **d** 21 or 22 **e** 480

16 **a** Togo **b** Sri Lanka
c Sri Lanka **d** Russian Federation
e $\frac{1}{261}$

9.3 Standard form without a calculator

Exercise 9C

1 **a** 9.6×10^{9} **b** 1.53×10^{10} **c** 4.8×10^{9}
d 9×10^{8} **e** 1.44×10^{10} **f** 4.05×10^{10}

2 **a** 3.73×10^{8} **b** 5.1×10^{7}

3 **a** 1.055×10^{6} km^2
b 4.3×10^{4} km^2

4 **a** 6.7×10^{9} **b** 3.69×10^{7} **c** 3.88×10^{8} **d** 3.8×10^{10}

5 **a** S = 5×10^{6} and N = 8×10^{7}
b **i** 4×10^{14} **ii** 2.5×10^{13}
iii 6.4×10^{15} **iv** 1.25×10^{20}

6 **a** Q **b** 2.75×10^{-3} **c** 2.25×10^{-3}

7 10^{3} cm^3 or 1000 cm^3 or 1 litre

8 **a** 1.5×10^{11} m
b 8 minutes 20 seconds

9 **a** 2.7×10^{3} **b** 3×10^{10}
c 8.1×10^{13} **d** 9×10^{-8}

10 1.82×10^{-22} kg

9.4 Surds

Exercise 9D

1 **a** $\sqrt{33}$ **b** $\sqrt{35}$ **c** 10
d 6 **e** 12 f 20

2 **a** $2\sqrt{2}$ **b** $2\sqrt{3}$ **c** $4\sqrt{2}$
d $5\sqrt{3}$ **e** $10\sqrt{2}$ **f** $7\sqrt{3}$

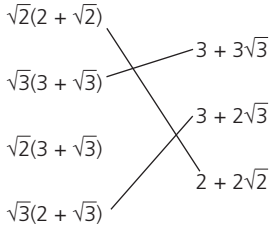
3 **a** $8\sqrt{2}$ **b** $2\sqrt{5}$ **c** $3\sqrt{7}$
d $2\sqrt{11}$ **e** $3\sqrt{5}$ **f** $9\sqrt{5}$

4 **a** $\sqrt{24}$ **b** $\sqrt{108}$ **c** $\sqrt{700}$
d $\sqrt{245}$ **e** $\sqrt{1000}$ **f** $\sqrt{2}$

5 **a** $5\sqrt{2}$ **b** $3\sqrt{3}$ **c** $9\sqrt{2}$
d $\sqrt{5}$ **e** $12\sqrt{3}$ **f** $7\sqrt{7}$

6 **a** $2 + 5\sqrt{2}$ **b** $3\sqrt{2} + 4$ **c** $3\sqrt{3} + 6$
d $10 - 3\sqrt{5}$ **e** $15\sqrt{2} - 12$ **f** 4

7 $\sqrt{2}(2 + \sqrt{2})$



The odd one out is: $\sqrt{2}(3 + \sqrt{3})$

8 **a** $\sqrt{8} + \sqrt{18} + \sqrt{32} = 2\sqrt{2} + 3\sqrt{2} + 4\sqrt{2} = 9\sqrt{2}$
b 128

9 **a** Three ways to do this: $8\sqrt{2}$ or $4\sqrt{8}$ or $2\sqrt{32}$
b $9\sqrt{3}$ or $3\sqrt{27}$

10 $(\sqrt{2} + 1)(\sqrt{2} - 1) = \sqrt{2}(\sqrt{2} - 1) + (\sqrt{2} - 1) = 2 - \sqrt{2} + \sqrt{2} - 1 = 1$

9.5 Rationalising the denominator

Exercise 9E

1 **a** $11 + 5\sqrt{5}$ **b** $5 - 2\sqrt{3}$ **c** $10 + 5\sqrt{6}$
d $4 + \sqrt{2}$ **e** $14 - 7\sqrt{2}$ **f** $11 + 4\sqrt{7}$

2 **a** 23 **b** 6 **c** 8 **d** 9 **e** 1 **f** 8

3 $3 + \sqrt{5}$ is a possible value of n. Multiples of this are also possible values.

4 **a** $4\sqrt{2}$ **b** $\frac{10}{3}\sqrt{3}$ **c** $\frac{3}{2}\sqrt{12}$ or $3\sqrt{3}$
d $4\sqrt{5}$ **e** 2 **f** $\frac{1}{2}$

5 **a** $\frac{2\sqrt{5}+5}{5}$ **b** $2\sqrt{6} - 1$
c $\frac{1}{2}(\sqrt{8} + 1)$ **d** $\frac{9-2\sqrt{3}}{3}$ or $\frac{1}{3}(9 - 2\sqrt{3})$

6 **a** $\frac{2}{7}(3 - \sqrt{2})$ **b** $\frac{5}{6}(4 + \sqrt{10})$ **c** $2\sqrt{3} - 3$ **d** $\frac{1}{4}(5 + 3\sqrt{5})$

7 **a** $\frac{1}{2+\sqrt{3}} = \frac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})} = \frac{2-\sqrt{3}}{4-3} = 2 - \sqrt{3}$

b $2 + \sqrt{3}$ **c** $\frac{1}{2}(3 - \sqrt{7})$

8 **a** $3 + 2\sqrt{2}$ **b** $\frac{\sqrt{3}}{3}$ **c** $\frac{7+\sqrt{5}}{4}$ **d** $-2 - 2\sqrt{12}$

9 **a** $(1+\sqrt{2})^2 - 2(1+\sqrt{2}) = 1 + 2\sqrt{2} + 2 - 2 - 2\sqrt{2} = 1$

b $(1-\sqrt{2})^2 - 2(1-\sqrt{2}) = 1 - 2\sqrt{2} + 2 - 2 + 2\sqrt{2} = 1$

Answers to Chapter 10

10.1 Units of measurement

Exercise 10A

1 **a** metres **b** kilometres
c millimetres **d** kilograms or grams
e litres **f** tonnes
g millilitres **h** metres
i kilograms **j** millimetres

2 Check individual answers.

3 The 5 metre since his height is about 175 cm, the lamp post will be about 525 cm

10.2 Converting between metric units

Exercise 10B

1 **a** 1.25 m **b** 8.2 cm **c** 0.55 m **d** 4.2 kg
e 5.75 t **f** 8.5 cl **g** 0.755 kg **h** 0.8 l
i 2 l **j** 1.035 m^3 **k** 0.53 m^3 **l** 34 000 m

2 **a** 3400 mm **b** 135 mm **c** 67 cm **d** 640 m
e 2400 ml **f** 590 cl **g** 3750 kg **h** 0.00094 l
i 2160 cl **j** 15 200 g **k** 14 000 l **l** 0.19 ml

3 He should choose the 2000 mm × 15 mm × 20 mm

4 1 000 000

5 400 hours

6 7.5×10^9

10.3 Time

Exercise 10C

1 **a** **i** $\frac{3}{4}$ **ii** $\frac{1}{3}$ **iii** $\frac{1}{12}$

b **i** $1\frac{2}{3}$ minutes or 1.67 minutes (to 2 d.p.)

ii $16\frac{2}{3}$ minutes or 16.67 minutes (to 2 d.p.)

c **i** 300 **ii** 3600

2 **a** 168 hours **b** 114 years

3 **a** 1 hour 10 minutes; 2 hours 3 minutes; 2 hours 9 minutes; 1 hour 45 minutes

b The 09:00

4 **a** 9:45 am, 10:36 am, 1:33 pm, 4:49 pm

b 3 hours and 48 minutes, 6 hours and 13 minutes

5 **a** 2 hours 50 minutes **b** 8 hours 58 minutes
c 6 hours 28 minutes

6 **a** 16:05 **b** 07:15 **c** 6 hours 45minutes

7 **a** 10:50 **b** 16:35 **c** 5 hours 45 minutes

8 **a** 12:10 **b** 2 hours 50 minutes

9 14:15

10 07:15 the next day

11 **a** 16:47 **b** Auckland is 3 hours ahead of Tokyo.
c Tokyo is 13 hours ahead of New York.
d The plane arrives in Tokyo at 13:55 on 26 June.
e

Today, –8HRS London	03:25
Today, +3HRS Auckland	14:25
Today, +0HRS Tokyo	11:25
Yesterday, –13HRS New York	22:25

10.4 Currency conversions

Exercise 10D

1 3197.41

2 164

3 The missing values are 4.30, 7.76, 38.78, 193.88, 387.75, 775.50

4 43.01

5 **a** 224.91 **b** 172.74

6 **a** **i** 349.83 **ii** 24 692 **iii** 432.90
b 54 000 yen, 500 euros, 650 dollars

7 **a** 2391.38 **b** 3489.75
c Taiwan dollar **d** 1.4593

8 **a** 74.7755 **b** 0.14747

10.5 Using a calculator efficiently

Exercise 10E

1 **a** 144 **b** 108

2 **a** 12.54 **b** 27.45

3 **a** 196.48 **b** 1.023 **c** 0.236 **d** 4.219

4 **a** 3.58 **b** 6

5 **a** 497.952 **b** 110.978625

6 **a** 3.12 **b** 0.749 **c** 90.5
d 185 **e** 5.56 **f** 27.5

Answers to Chapter 11

11.1 The language of algebra

Exercise 11A

1 **a** $x + 2$ **b** $x - 6$ **c** $x + k$ **d** $x - t$
e $x + 3$ **f** $d + m$ **g** $b - y$ **h** $p + t + w$
i $8x$ **j** hj **k** $x \div 4$ or $\frac{x}{4}$ **l** $2 \div x$ or $\frac{2}{x}$
m $y \div t$ or $\frac{y}{t}$ **n** wt **o** a^2 **p** g^2

2 **a** $x + 3$ years **b** $x - 4$ years

3 $F = 2C + 30$

4 Rule **c**

5 **a** $3n$ **b** $3n + 3$ **c** $n + 1$ **d** $n - 1$

6 Anil: $2n$, Reza: $n + 2$, Dale: $n - 3$, Chen: $2n + 3$

7 **a** \$4 **b** $\$(10 - x)$ **c** $\$(y - x)$ **d** $\$2x$

8 **a** \$75 **b** $\$15x$ **c** \$4A **d** $\$Ay$

9 $(A - B)$ dollars

10 $\$A \div 5$ or $\$\frac{A}{5}$.

11 **a** Dad: $(72 + x)$ years, me: $(T + x)$ years **b** 31 years

12 **a** $T \div 2$ or $\frac{T}{2}$ **b** $T \div 2 + 4$ or $\frac{T}{2} + 4$ **c** $T - x$

13 **a** $8x$ **b** $12m$ **c** $18t$

14 Andrea: $3n - 3$, Barak: $3n - 1$, Ahmed: $3n - 6$ or $3(n - 2)$, Dina: 0, Emma: $3n - n = 2n$. Hana: $3n - 3m$

15 For example, $2 \times 6m$, $1 \times 12m$, $6m + 6m$, etc.

11.2 Substitution into formulas

Exercise 11B

1 **a** 8 **b** 17 **c** 32

2 **a** 3 **b** 11 **c** 43

3 **a** 9 **b** 15 **c** 29

4 **a** 9 **b** 5 **c** –1

5 **a** 13 **b** 33 **c** 78

6 **a** 10 **b** 13 **c** 58

7 **a** \$4 **b** 13 km **c** Yes, the fare is \$5.00

8 **a** $2 \times 8 + 6 \times 11 - 3 \times 2 = 76$
b $5 \times 2 - 2 \times 11 + 3 \times 8 = 12$

9 Any values such that $lw = \frac{1}{2}bh$ or $bh = 2lw$

10 **a** 32 **b** 64 **c** 160

11 **a** 6.5 **b** 0.5 **c** –2.5

12 **a** 2 **b** 8 **c** –10

13 **a** 3 **b** 2.5 **c** –5

14 **a** 6 **b** 3 **c** 2

15 **a** 12 **b** 8 **c** $1\frac{1}{2}$

16 **a** $\frac{1050}{n}$ **b** \$925

17 **a** **i** odd **ii** odd **iii** even **iv** odd
b Any valid expression such as $xy + z$

18 **a** \$20
b **i** –\$40 **ii** Delivery cost will be zero. **c** 40 kilometres

11.3 Rearranging formulas

Exercise 11C

1 $k = \frac{T}{3}$

2 $y = X + 1$

3 $p = 3Q$

4 $r = \frac{A - 9}{4}$

5 $n = \frac{W + 1}{3}$

6 **a** $m = p - t$
b $t = p - m$

7 $m = gv$

8 $r = \frac{C}{2\pi}$

9 $b = \frac{A}{h}$

10 $l = \frac{P - 2w}{2}$

11 **a** $-40 - 32 = -72$, $-72 \div 9 = -8$, $5 \times -8 = -40$
b $68 - 32 = 36$, $36 \div 9 = 4$, $4 \times 5 = 20$
c student's own demonstration

12 **a** $a = \frac{v - u}{t}$ **b** $t = \frac{v - u}{a}$

13 **a** $n = \frac{W - t}{3}$ **b** $t = W - 3n$

14 **a** $y = \frac{x + w}{5}$ **b** $w = 5y - x$

15 **a** $b = ac - 2$ **b** $c = \frac{b + 2}{a}$

16 $t = \frac{r}{p} + 3$

17 $T = \frac{b^2 + c^2 - a^2}{2bc}$

11.4 More complicated formulas

Exercise 11D

1 $m = \sqrt{t}$

2 $p = \sqrt{m - 2}$

3 $d = \sqrt{\frac{4A}{\pi}}$

4 $p = \sqrt{\frac{k}{2}}$

5 **a** $t = u^2 - v$ **b** $u = \sqrt{v + t}$

6 **a** $m = k - n^2$ **b** $n = \sqrt{k - m}$

7 $r = \sqrt{\frac{T}{5}}$

8 **a** $w = K - 5n^2$ **b** $n = \sqrt{\frac{K - w}{5}}$

9 **a** 2.5 **b** $a = \sqrt{c^2 - b^2}$

10 **a** 60 **b** $a = \frac{2(s - ut)}{t^2}$

11 $e = \left(\frac{12}{d} - 1\right)^2$

12 **a** 5 **b** $u = \sqrt{v^2 - 2as}$ **c** $s = \frac{v^2 - u^2}{2a}$

13 **a** $L = \left(\frac{T}{2\pi}\right)^2 G$ **b** Student's proof

14 **a** $R = \sqrt{\frac{D + \pi r^2}{\pi}}$ **b** $r = \sqrt{\frac{\pi R^2 - D}{\pi}}$ **c** $\pi = \frac{D}{R^2 - r^2}$

15 **a** $x = 5$ or -5 **b** $x = \sqrt{\frac{11 + 4y^2}{3}}$ **c** $y = \sqrt{\frac{3x^2 - 11}{4}}$

16 **a** $a = \left(\frac{T}{2}\right)^2 (c + 3)$ **b** $c = a\left(\frac{2}{T}\right)^2 - 3$

17 **a** 12 **b** $f = \frac{uv}{u + v}$
c $u = \frac{fv}{v - f}$ **d** $v = \frac{fu}{u - f}$

Answers to Chapter 12

12.1 Simplifying expressions

Exercise 12A

1 **a** $6t$ **b** $15y$ **c** $8w$ **d** $5b^2$ **e** $2w^2$
f $8p^2$ **g** $6t^2$ **h** $15t^2$ **i** $2mt$ **j** $5qt$
k $6mn$ **l** $6qt$ **m** $10hk$ **n** $21pr$

2 **a** All except $2m \times 6m$ **b** 2 and 0

3 $4x$ cm

4 **a** y^3 **b** $3m^3$ **c** $4t^3$ **d** $6n^3$ **e** t^4
f h^5 **g** $12n^5$ **h** $6a^7$ **i** $4k^7$ **j** t^3
k $12d^3$ **l** $15p^6$ **m** $3mp^2$ **n** $6m^2n$ **o** $8m^2p^2$

Exercise 12B

1 **a** $\$t$ **b** $\$(4t + 3)$

2 **a** $10x + 2y$ **b** $7x + y$ **c** $6x + y$

3 **a** $5a$ **b** $6c$ **c** $9e$ **d** $6f$
e $4j$ **f** $3q$ **g** 0 **h** $-w$
i $6x^2$ **j** $5y^2$ **k** 0

4 **a** $7x$ **b** $3t$ **c** $-5x$ **d** $-5k$
e $2m^2$ **f** 0

5 **a** $7x + 5$ **b** $5x + 6$ **c** $5p$ **d** $5x + 6$
e $5p + t + 5$ **f** $8w - 5k$ **g** c **h** $8k - 6y + 10$

6 **a** $2c + 3d$ **b** $5d + 2e$ **c** $f + 3g + 4h$
d $6u - 3v$ **e** $7m - 7n$ **f** $3k + 2m + 5p$
g $2v$ **h** $2w - 3y$ **i** $11x^2 - 5y$
j $-y^2 - 2z$ **k** $x^2 - z^2$

7 **a** $8x + 6$ **b** $3x + 16$ **c** $2x + 2y + 8$

8 Any acceptable answers, e.g. $x + 4x + 2y + 2y$ or $6x - x + 6y - 2y$

9 **a** $2x$ and $2y$ **b** a and $7b$

10 **a** $3x - 1 - x$ **b** $10x$ **c** 25 cm

11 Maria is correct, as the two short horizontal lengths are equal to the bottom length and the two short vertical lengths are equal to the side length.

12.2 Expanding brackets

Exercise 12C

1 **a** $6 + 2m$ **b** $10 + 5l$ **c** $12 - 3y$
d $20 + 8k$ **e** $6 - 12f$ **f** $10 - 6w$
g $10k + 15m$ **h** $12d - 8n$ **i** $t^2 + 3t$
j $k^2 - 3k$ **k** $4t^2 - 4t$ **l** $8k - 2k^2$
m $8g^2 + 20g$ **n** $15h^2 - 10h$ **o** $y^3 + 5y$
p $h^4 + 7h$ **q** $k^3 - 5k$ **r** $3t^3 + 12t$
s $15d^3 - 3d^4$ **t** $6w^3 + 3tw$ **u** $15a^3 - 10ab$
v $12p^4 - 15mp$ **w** $12h^3 + 8h^2g$ **x** $8m^3 + 2m^4$

2 **a** $5(t - 1)$ and $5t - 5$
b Yes, as $5(t - 1)$ when $t = 4.50$ is $5 \times 3.50 = \$17.50$

3 He has worked out 3×5 as 8 instead of 15 and he has not multiplied the second term by 3. Answer should be $15x - 12$.

4 **a** $3(2y + 3)$ **b** $2(6z + 4)$ or $4(3z + 2)$

Exercise 12D

1 **a** $7t$ **b** $9d$ **c** $3e$ **d** $2t$
e $5t^2$ **f** $4y^2$ **g** $5ab$ **h** $3a^2d$

2 **a** $2x$ and $11y$ **b** a and $8b$

3 **a** $2x - 3$ **b** $10x - 16$ or $2(5x - 8)$

4 **a** $22 + 5t$ **b** $21 + 19k$ **c** $22 + 2f$ **d** $14 + 3g$

5 **a** $2 + 2h$ **b** $9g + 5$ **c** $17k + 16$ **d** $6e + 20$

6 **a** $4m + 3p + 2mp$ **b** $3k + 4h + 5hk$
c $12r + 24p + 13pr$ **d** $19km + 20k - 6m$

7 **a** $9t^2 + 13t$ **b** $13y^2 + 5y$ **c** $10e^2 - 6e$ **d** $14k^2 - 3kp$

8 **a** $17ab + 12ac + 6bc$ **b** $18wy + 6ty - 8tw$
c $14mn - 15mp - 6np$ **d** $8r^3 - 6r^2$

9 For x-coefficients, 3 and 1 or 1 and 4; for y-coefficients, 5 and 1 or 3 and 4 or 1 and 7.

10 $5(3x + 2) - 3(2x - 1) = 9x + 13$

12.3 Factorisation

Exercise 12E

1 **a** $6(m + 2t)$ **b** $3(3t + p)$ **c** $4(2m + 3k)$
d $4(r + 2t)$ **e** $m(n + 3)$ **f** $g(5g + 3)$
g $2(2w - 3t)$ **h** $y(3y + 2)$ **i** $t(4t - 3)$
j $3m(m - p)$ **k** $3p(2p + 3t)$ **l** $2p(4t + 3m)$
m $4b(2a - c)$ **n** $5bc(b - 2)$ **o** $2b(4ac + 3de)$
p $2(2a^2 + 3a + 4)$ **q** $3b(2a + 3c + d)$ **r** $t(5t + 4 + a)$
s $3mt(2t - 1 + 3m)$ **t** $2ab(4b + 1 - 2a)$ **u** $5pt(2t + 3 + p)$

2 **a** Suni has taken out a common factor.
b Because the bracket adds up to \$10.
c \$30

3 **a**, **d**, **f** and **h** do not factorise.
b $m(5 + 2p)$ **c** $t(t - 7)$ **e** $2m(2m - 3p)$
g $a(4a - 5b)$ **i** $b(5a - 3bc)$

4 **a** Bernice
b Ahmed has not taken out the largest possible common factor. Craig has taken m out of both terms but there isn't an m in the second term.

5 There are no common factors.

6 numerator $4x^2 - 12x$, denominator $2x - 6$

7 **a** $4(x + 1)$ **b** $2(x + 4)$ **c** $4(x + 1)$ **d** $2(3x + 2$

8 **a** **i** 175 **ii** 6125 **b** **i** ab^2 **ii** a^2b^3

9 **a** a^2c **b** a^3bc^2

10 **a** $3de$ **b** $6d^2e^3f$

11 **a** $4x$ **b** $24x^2y^2z$

12 **a** $6(x + 2y)$ **b** $2x(x + 2y)$ **c** $3(x + 2y)$ **d** $6x(x + 2y)$

13 **a** $2x - 1$ **b** $6x(2x - y)$

12.4 Multiplying two brackets: 1

Exercise 12F

1 $x^2 + 5x + 6$
2 $t^2 + 7t + 12$
3 $w^2 + 4w + 3$
4 $m^2 + 6m + 5$
5 $k^2 + 8k + 15$
6 $a^2 + 5a + 4$
7 $x^2 + 2x - 8$
8 $t^2 + 2t - 15$
9 $w^2 + 2w - 3$
10 $f^2 - f - 6$
11 $g^2 - 3g - 4$
12 $y^2 + y - 12$
13 $x^2 + x - 12$
14 $p^2 - p - 2$
15 $k^2 - 2k - 8$
16 $y^2 + 3y - 10$
17 $a^2 + 2a - 3$
18 $x^2 - 9$
19 $t^2 - 25$
20 $m^2 - 16$
21 $t^2 - 4$
22 $y^2 - 64$
23 $p^2 - 1$
24 $25 - x^2$
25 $49 - g^2$
26 $x^2 - 36$
27 $(x + 2)$ and $(x + 3)$

28 **a** B: $1 \times (x - 2)$ C: 1×2 D: $2 \times (x - 1)$
b $(x - 2) + 2 + 2(x - 1) = 3x - 2$
c Area A = $(x - 1)(x - 2)$ = area of square minus areas (B + C + D)
$= x^2 - (3x - 2)$
$= x^2 - 3x + 2$

29 **a** $x^2 - 9$
b **i** 9991 **ii** 39991

12.5 Multiplying two brackets: 2

Exercise 12G

1 $6x^2 + 11x + 3$
2 $12y^2 + 17y + 6$
3 $6t^2 + 17t + 5$
4 $8t^2 + 2t - 3$
5 $10m^2 - 11m - 6$
6 $12k^2 - 11k - 15$
7 $6p^2 + 11p - 10$
8 $10w^2 + 19w + 6$
9 $6a^2 - 7a - 3$
10 $8r^2 - 10r + 3$
11 $15g^2 - 16g + 4$
12 $12d^2 + 5d - 2$
13 $8p^2 + 26p + 15$
14 $6t^2 + 7t + 2$
15 $6p^2 + 11p + 4$
16 $-10t^2 - 7t + 6$
17 $-6n^2 + n + 12$
18 $6f^2 - 5f - 6$
19 $-10q^2 + 7q + 12$
20 $-6p^2 - 7p + 3$
21 $-6t^2 + 10t + 4$

22 **a** $x^2 - 1$ **b** $4x^2 - 1$ **c** $4x^2 - 9$
d $9x^2 - 25$

23 **a** $(3x - 2)(2x + 1) = 6x^2 - x - 2$
$(2x - 1)(2x - 1) = 4x^2 - 4x + 1$
$(6x - 3)(x + 1) = 6x^2 + 3x - 3$
$(3x + 2)(2x + 1) = 6x^2 + 7x + 2$
b Multiply the x terms to match the x^2 term and/or multiply the constant terms to get the constant term in the answer.

Exercise 12H

1 $4x^2 - 1$
2 $9t^2 - 4$
3 $25y^2 - 9$
4 $16m^2 - 9$
5 $4k^2 - 9$
6 $16h^2 - 1$

7 $4 - 9x^2$
8 $25 - 4t^2$
9 $36 - 25y^2$
10 $a^2 - b^2$
11 $9t^2 - k^2$
12 $4m^2 - 9p^2$
13 $25k^2 - g^2$
14 $a^2b^2 - c^2d^2$
15 $a^4 - b^4$
16 **a** $a^2 - b^2$
b Dimensions: $a + b$ by $a - b$; Area: $a^2 - b^2$
c Areas are the same, so $a^2 - b^2 = (a + b) \times (a - b)$
17 First shaded area is $(2k)^2 - 1^2 = 4k^2 - 1$
Second shaded area is $(2k + 1)(2k - 1) = 4k^2 - 1$

Exercise 12I

1 $x^2 + 10x + 25$
2 $m^2 + 8m + 16$
3 $t^2 + 12t + 36$
4 $p^2 + 6p + 9$
5 $m^2 - 6m + 9$
6 $t^2 - 10t + 25$
7 $m^2 - 8m + 16$
8 $k^2 - 14k + 49$
9 $9x^2 + 6x + 1$
10 $16t^2 + 24t + 9$
11 $25y^2 + 20y + 4$
12 $4m^2 + 12m + 9$
13 $16t^2 - 24t + 9$
14 $9x^2 - 12x + 4$
15 $25t^2 - 20t + 4$
16 $25r^2 - 60r + 36$
17 $x^2 + 2xy + y^2$
18 $m^2 - 2mn + n^2$
19 $4t^2 + 4ty + y^2$
20 $m^2 - 6mn + 9n^2$
21 $x^2 + 4x$
22 $x^2 - 10x$
23 $x^2 + 12x$
24 $x^2 - 4x$
25 **a** Marcela has just squared the first term and the second term. She hasn't written down the brackets twice.
b Paulo has written down the brackets twice but has worked out $(3x)^2$ as $3x^2$ and not $9x^2$.
c $9x^2 + 6x + 1$
26 Whole square is $(2x)^2 = 4x^2$.
Three areas are $2x - 1$, $2x - 1$ and 1.
$4x^2 - (2x - 1 + 2x - 1 + 1) = 4x^2 - (4x - 1) = 4x^2 - 4x + 1$

12.6 Expanding three brackets

Exercise 12J

1 **a** $x^2 + 2x - 3$ **b** $x^3 + 2x^2 - 3x$ **c** $x^3 + 4x^2 + x - 6$
2 **a** $x^2 - 7x + 10$ **b** $x^3 - 6x^2 + 3x - 10$
c $2x^3 - 13x^2 + 13x + 10$
3 **a** $x^3 - 3x^2 - 13x + 15$ **b** $3x^3 + 31x^2 + 78x + 56$
c $x^3 - 14x^2 + 53x - 40$
4 **a** $x^2 + 4x + 4$ **b** $x^3 + 6x^2 + 12x + 8$
c $8x^3 + 12x^2 + 6x + 1$
5 **a** $x^3 + x^2 - 4x - 4$ **b** $2x^3 - 3x^2 - 11x + 6$
c $x^3 + 4x^2 - 4x - 16$
6 **a** $x^3 + 6x^2 + 11x + 6$ **b** $x^3 - 6x^2 + 11x - 6$
7 **a** $x^3 + 4x^2 - 3x - 18$ **b** $x^3 - 6x^2 - 15x + 100$
c $9x^3 + 78x^2 - 116x + 40$
8 **a** $(x + 1)^3 - (x - 1)^3 = x^3 + 3x^2 + 3x + 1 - (x^3 - 3x^2 + 3x - 1)$
$= x^3 + 3x^2 + 3x + 1 - x^3 + 3x^2 - 3x + 1$
$= 6x^2 + 2 = 2(3x^2 + 1)$
b $4(3x^2 + 4)$
9 The volume of the cube is $(x + 1)^3$
One of the eight pieces is a cube of side x and volume x^3
Three of the eight pieces are cuboids, with sides x, x and 1 and each has volume x^2
Three of the eight pieces are cuboids with sides x, 1 and 1 and each has volume x
One of the eight pieces is a cube of side 1 and volume 1
Add these eight volumes to get $x^3 + 3x^2 + 3x + 1$ which is $(x + 1)^3$
10 $a = 6$, $b = 5$, $c = -8$
11 **a** $x^3 - 1$ **b** $x^3 - 8$
c $x^3 - 27 = (x^2 + 3x + 9)(x - 3)$
12 $6x^3 + 11x^2 + 6x + 1$ cm^3

12.7 Quadratic factorisation

Exercise 12K

1 $(x + 2)(x + 3)$
2 $(t + 1)(t + 4)$
3 $(m + 2)(m + 5)$
4 $(k + 4)(k + 6)$
5 $(p + 2)(p + 12)$
6 $(r + 3)(r + 6)$
7 $(w + 2)(w + 9)$
8 $x(x + 3)\,(x + 4)$
9 $a(a + 6)\,(a + 2)$
10 $k(k + 3)\,(k + 7)$
11 $(f + 1)(f + 21)$
12 $(b + 8)(b + 12)$
13 $(t - 2)(t - 3)$
14 $(d - 4)(d - 1)$
15 $(g - 2)(g - 5)$
16 $(x - 3)(x - 12)$
17 $(c - 2)(c - 16)$
18 $t(t - 4)\,(t - 9)$
19 $(y - 4)(y - 12)$
20 $(j - 6)(j - 8)$
21 $p(p - 5)\,(p - 3)$
22 $(y + 6)(y - 1)$
23 $(t + 4)(t - 2)$
24 $(x + 5)(x - 2)$
25 $(m + 2)(m - 6)$

26 $(r + 1)(r - 7)$
27 $(n + 3)(n - 6)$
28 $(m + 4)(m - 11)$
29 $(w + 4)(w - 6)$
30 $(t + 9)(t - 10)$
31 $h(h - 9)(h + 8)$
32 $(t + 7)(t - 9)$
33 $(d + 1)^2$
34 $(y + 10)^2$
35 $t(t - 4)^2$
36 $(m - 9)^2$
37 $(x - 12)^2$
38 $(d + 3)(d - 4)$
39 $(t + 4)(t - 5)$
40 $(q + 7)(q - 8)$
41 $(x + 2)(x + 3)$, giving areas of $2x$ and $3x$, or $(x + 1)(x + 6)$, giving areas of x and $6x$.

Exercise 12L

1 $(x + 3)(x - 3)$
2 $(t + 5)(t - 5)$
3 $(m + 4)(m - 4)$
4 $(3 + x)(3 - x)$
5 $(7 + t)(7 - t)$
6 $(k + 10)(k - 10)$
7 $(2 + y)(2 - y)$
8 $(x + 8)(x - 8)$
9 $(t + 9)(t - 9)$
10 **a** x^2
b **i** $(x - 2)$ **ii** $(x + 2)$ **iii** $x(x - 2) = x^2 - 2x$ **iv** 4
c A + B − C = $x^2 - 4$, which is the area of D, which is $(x + 2)(x - 2)$.
11 **a** $x^2 + 4x + 4 - (x^2 + 2x + 1) = 2x + 3$
b $(a + b)(a - b)$
c $(x + 2 + x + 1)(x + 2 - x - 1) = (2x + 3)(1) = 2x + 3$
d The answers are the same.
e $(x + 1 + x - 1)(x + 1 - x + 1) = (2x)(2) = 4x$
12 $(x + y)(x - y)$
13 $(x + 2y)(x - 2y)$
14 $(x + 3y)(x - 3y)$
15 $(3x + 1)(3x - 1)$
16 $(4x + 3)(4x - 3)$
17 $(5x + 8)(5x - 8)$
18 $(2x + 3y)(2x - 3y)$
19 $(3t + 2w)(3t - 2w)$
20 $(4y + 5x)(4y - 5x)$
21 **a** $(x - 3)(x + 3)$ **b** $x(x - 3)(x + 3)$
22 **a** $(x - 6)(x + 6)$ **b** $x(x - 6)(x + 6)$
23 **a** $x(x + 1)(x - 1)$ **b** $x(x + 2)(x - 2)$ **c** $x(x + 10)(x - 10)$
24 **a** $x(x + 5)(x - 5)$ **b** $2x(x + 5)(x - 5)$ **c** $3x(x + 5)(x - 5)$

Exercise 12M

1 $(2x + 1)(x + 2)$
2 $(7x + 1)(x + 1)$
3 $(4x + 7)(x - 1)$
4 $(3t + 2)(8t + 1)$
5 $(3t + 1)(5t - 1)$
6 $(4x - 1)^2$
7 $3(y + 7)(2y - 3)$
8 $4(y + 6)(y - 4)$
9 $(2x + 3)(4x - 1)$
10 $(2t + 1)(3t + 5)$
11 $(x - 6)(3x + 2)$
12 $(x - 5)(7x - 2)$
13 **a** $(x + 5)^2 - 25$
b **i** $(x + 5)^2 - 15$ **ii** $(x + 5)^2 - 5$ **iii** $(x + 5)^2$
c $(x - 5)^2 - 25$
14 **a** $(x + 1)^2 + 1$ **b** $(x - 1)^2 + 4$ **c** $(x + 4)^2 - 27$
d $(x - 2)^2 - 7$ **e** $(x + 2)^2 + 6$ **f** $(x - 3)^2$
15 **a** $2(x + 1)^2 + 7$ **b** $3(x - 2)^2 + 8$
c $10(x - 5)^2 - 25$ **d** $4(x + 1)^2 - 1$
16 **a** $(x - 1.5)^2 + 2.75 = x^2 - 3x + 2.25 + 2.75 = x^2 - 3x + 4$
b **i** $(x + 0.5)^2 + 0.75$
ii $(x - 0.5)^2 - 1.25$
iii $(x - 2.5)^2 + 2.25$
17 $4x + 1$ and $3x + 2$
18 **a** All the terms in the quadratic have a common factor of 6.
b $6(x + 2)(x + 3)$. This has the highest common factor taken out.
c For example, 'A rectangle could be split in many different ways.'
19 **a** $x(2x + 1)(x + 3)$ **b** $x(2x - 1)(x + 2)$ **c** $x(3x - 1)(x - 2)$
d $x(2x - 1)^2$ **e** $x(3x + 2)(x - 2)$ **f** $2x(x + 1)(x + 2)$

12.8 Algebraic fractions

Exercise 12N

1 **a** $\frac{5x}{6}$ **b** $\frac{19x}{20}$ **c** $\frac{23x}{20}$ **d** $\frac{3x + 2y}{6}$
e $\frac{x^2y + 8}{4x}$ **f** $\frac{5x + 7}{6}$ **g** $\frac{7x + 3}{4}$ **h** $\frac{13x + 5}{15}$
i $\frac{3x - 7}{4}$ **j** $\frac{5x - 10}{4}$

2 **a** $\frac{x}{6}$ **b** $\frac{11x}{20}$ **c** $\frac{7x}{20}$ **d** $\frac{3x - 2y}{6}$
e $\frac{xy^2 - 8}{4y}$ **f** $\frac{x - 1}{6}$ **g** $\frac{x + 1}{4}$ **h** $\frac{-7x - 5}{15}$
i $\frac{x - 1}{4}$ **j** $\frac{2 - 3x}{4}$

3 **a** $\frac{x^2}{6}$ **b** $\frac{3xy}{14}$ **c** $\frac{8}{3}$ **d** $\frac{2xy}{3}$
e $\frac{x^2 - 2x}{10}$ **f** $\frac{1}{6}$ **g** $\frac{6x^2 + 5x + 1}{8}$ **h** $\frac{2x^2 + x}{15}$
i $\frac{2x - 4}{x - 3}$ **j** $\frac{1}{2x}$

4 **a** x **b** $\frac{x}{2}$ **c** $\frac{3x^2}{16}$ **d** 3
e $\frac{17x + 1}{10}$ **f** $\frac{13x + 9}{10}$ **g** $\frac{3x^2 - 5x - 2}{10}$ **h** $\frac{x + 3}{2}$
i $\frac{2x^2 - 6y^2}{9}$

5 **a** $\frac{7x+9}{(x+1)(x+2)}$ **b** $\frac{11x-10}{(x-2)(x+1)}$ **c** $\frac{2-13x}{(4x+1)(x+2)}$
d $\frac{8-10x}{(2x-1)(x+1)}$ **e** $\frac{x+1}{(2x-1)(3x-1)}$

6 First, he did not factorise and just cancelled the x^2s. Then he cancelled 2 and 6 with the wrong signs. Then he said two minuses make a plus when adding, which is not true.

7 $\frac{2x^2+x-3}{4x^2-9}$

8 **a** $\frac{9x+13}{(x+1)(x+2)}$ **b** $\frac{14x+19}{(4x-1)(x+1)}$ **c** $\frac{2x^2+x-13}{2(x+1)}$ **d** $\frac{x+1}{(2x-1)(3x-1)}$

9 **a** $\frac{x-1}{2x+1}$ **b** $\frac{2x+1}{x+3}$ **c** $\frac{2x-1}{3x-2}$ **d** $\frac{x+1}{x-1}$ **e** $\frac{2x+5}{4x-1}$

Answers to Chapter 13

13.1 Solving linear equations

Exercise 13A

1 **a** 56 **b** 2 **c** 6 **d** 3 **e** 4
f $2\frac{1}{2}$ **g** $3\frac{1}{2}$ **h** $2\frac{1}{2}$ **i** 4 **j** 21
k 72 **l** 56 **m** 0 **n** –7 **o** –18
p 36 **q** 36 **r** 60 **s** 7 **t** 11
u 2 **v** 7 **w** 2.8 **x** 1 **y** 11.5
z 0.2

2 **a** –4 **b** 15

3 **a** Elif
b Second line: Mustafa subtracts 1 instead of adding 1; fourth line: Mustafa subtracts 2 instead of dividing by 2.

Exercise 13B

1 **a** 3 **b** 7 **c** 5 **d** 3 **e** 4 **f** 6
g 8 **h** 1 **i** $1\frac{1}{2}$ **j** $2\frac{1}{2}$ **k** $\frac{1}{2}$ **l** $1\frac{1}{5}$
m 2 **n** –2 **o** –1 **p** –2 **q** –2 **r** –1

2 Any values that work, e.g. $a = 2$, $b = 3$ and $c = 30$.

3 55

Exercise 13C

1 **a** $x = 2$ **b** $y = 1$ **c** $a = 7$ **d** $t = 4$
e $p = 2$ **f** $k = -1$ **g** $m = 3$ **h** $s = -2$

2 $3x - 2 = 2x + 5$, $x = 7$

3 **a** $d = 6$ **b** $x = 11$ **c** $y = 1$ **d** $h = 4$
e $b = 9$ **f** $c = 6$

4 $6x + 3 = 6x + 10$; $6x - 6x = 10 - 3$; $0 = 7$, which is obviously false. Both sides have $6x$, which cancels out.

5 Check student's example.

13.2 Setting up equations

Exercise 13D

1 90 cents or 0.90 dollars

2 **a** 1.5 **b** 2

3 **a** 1.5 cm **b** 6.75 cm^2

4 17

5 8

6 **a** $8c - 10 = 56$ **b** $8.25

7 **a** B: 450 cars, C: 450 cars, D: 300 cars
b 800 **c** 750

8 360 dollars

9 3 years

10 9 years

11 3 cm

12 5

13 **a** $4x + 40 = 180$ **b** $x = 35°$

14 **a** $\frac{x+10}{5} = 9.50$ **b** $37.50

15 No, as $x + x + 2 + x + 4 + x + 6 = 360$ gives $x = 87°$ so the consecutive numbers (87, 89, 91, 93) are not even but odd

16 $4x + 18 = 3x + 1 + 50$, $x = 33$. Large bottle 1.5 litres, small bottle 1 litre

13.3 Solving quadratic equations by factorisation

Exercise 13E

1 –2, –5
2 –3, –1
3 –6, –4
4 –3, 2
5 –1, 3
6 –4, 5
7 1, –2
8 2, –5
9 7, –4
10 3, 2
11 1, 5
12 4, 3
13 –4, –1
14 –9, –2
15 2, 4
16 3, 5
17 –2, 5
18 –3, 5
19 –6, 2
20 –6, 3
21 –1, 2
22 –2
23 –5
24 4
25 –2, –6

26 7

27 **a** $x(x-3) = 550$, $x^2 - 3x - 550 = 0$

b $(x-25)(x+22) = 0$, $x = 25$ years

28 $x(x+40) = 48000$, $x^2 + 40x - 48000 = 0$, $(x+240)(x-200) = 0$. Fence is $2 \times 200 + 2 \times 240 = 880$ m.

29 –6, –4

30 2, 16

31 –6, 4

32 –9, 6

33 –10, 3

34 –4, 11

35 –8, 9

36 8, 9

37 1

38 Mario was correct. Sylvan did not make it into a standard quadratic and only factorised the x terms. She also incorrectly solved the equation $x - 3 = 4$.

Exercise 13F

1 **a** $\frac{1}{3}$, –3 **b** $1\frac{1}{3}$, $-\frac{1}{2}$ **c** $-\frac{1}{5}$, 2

d $-2\frac{1}{2}$, $3\frac{1}{2}$ **e** $-\frac{1}{6}$, $-\frac{1}{3}$ **f** $\frac{2}{3}$, 4

g $\frac{1}{2}$, –3 **h** $\frac{5}{2}$, $-\frac{7}{6}$ **i** $-1\frac{2}{3}$, $1\frac{2}{5}$

j $1\frac{3}{4}$, $1\frac{2}{7}$ **k** $\frac{2}{3}$, $\frac{1}{8}$ **l** $\pm\frac{1}{4}$

m $-2\frac{1}{4}$, 0 **n** $\pm 1\frac{2}{5}$ **o** $-\frac{1}{3}$, 3

2 **a** –6, 7 **b** $-\frac{5}{2}$, $\frac{3}{2}$ **c** –6, 7

d –1, $\frac{11}{13}$ **e** –2, 3 **f** $-\frac{2}{5}$, $\frac{1}{2}$

g $-\frac{1}{2}$, $-\frac{1}{3}$ **h** –2, $\frac{1}{5}$ **i** 4

j –2, $\frac{1}{8}$ **k** $-\frac{1}{3}$, 0 **l** –5, 5

m $-\frac{5}{3}$ **n** $-\frac{7}{2}$, $\frac{7}{2}$ **o** $-\frac{5}{2}$, 3

3 **a** Both have only one solution: $x = 1$.

b B is a linear equation, but A and C are quadratic equations.

4 **a** $(5x-1)^2 = (2x+3)^2 + (x+1)^2$, when expanded and collected into the general quadratics, gives the required equation.

b $(10x+3)(2x-3)$, $x = 1.5$; area = 7.5 cm^2.

13.4 Solving quadratic equations by the quadratic formula

Exercise 13G

1 1.77, –2.27

2 –0.23, –1.43

3 3.70, –2.70

4 0.29, –0.69

5 –0.19, –1.53

6 –1.23, –2.43

7 –0.41, –1.84

8 –1.39, –2.27

9 1.37, –4.37

10 2.18, 0.15

11 –0.39, –5.11

12 0.44, –1.69

13 1.64, 0.61

14 0.36, –0.79

15 1.89, 0.11

16 13

17 $x^2 - 3x - 7 = 0$

18 Hasan gets $x = \frac{4 \pm \sqrt{0}}{8}$ and Miriam gets $(2x-1)^2 = 0$; each method only gives one solution, $x = \frac{1}{2}$

13.5 Solving quadratic equations by completing the square

Exercise 13H

1 **a** $(x+2)^2 - 4$ **b** $(x+7)^2 - 49$

c $(x-3)^2 - 9$ **d** $(x+3)^2 - 9$

e $(x-1.5)^2 - 2.25$ **f** $(x-4.5)^2 - 20.25$

g $(x+6.5)^2 - 42.25$ **h** $(x+5)^2 - 25$

i $(x+4)^2 - 16$ **j** $(x-1)^2 - 1$

k $(x+1)^2 - 1$

2 **a** $(x+2)^2 - 5$ **b** $(x+7)^2 - 54$

c $(x-3)^2 - 6$ **d** $(x+3)^2 - 2$

e $(x-1.5)^2 - 3.25$ **f** $(x+3)^2 - 6$

g $(x-4.5)^2 - 10.25$ **h** $(x+6.5)^2 - 7.25$

i $(x+4)^2 - 22$ **j** $(x+1)^2 - 2$

k $(x-1)^2 - 8$ **l** $(x+1)^2 - 10$

3 **a** $-2 \pm \sqrt{5}$ **b** $-7 \pm 3\sqrt{6}$ **c** $3 \pm \sqrt{6}$

d $-3 \pm \sqrt{2}$ **e** $1.5 \pm \sqrt{3.25}$ **f** $-3 \pm \sqrt{6}$

g $4.5 \pm \sqrt{10.25}$ **h** $-6.5 \pm \sqrt{7.25}$ **i** $-4 \pm \sqrt{22}$

j $-1 \pm \sqrt{2}$ **k** $1 \pm 2\sqrt{2}$ **l** $-1 \pm \sqrt{10}$

4 **a** 1.45, –3.45 **b** 5.32, –1.32 **c** –4.16, 2.16

5 **a** $x = 1.5 \pm \sqrt{3.75}$ **b** $x = 1 \pm \sqrt{0.75}$

c $x = -1.25 \pm \sqrt{6.5625}$ **d** $x = 7.5 \pm \sqrt{40.25}$

6 $p = -14$, $q = -3$

7 3rd, 1st, 4th and 2nd – in that order

13.6 Fractional equations

Exercise 13I

1 **a** 4.8 **b** 60 **c** $\frac{24}{7}$ or $3\frac{3}{7}$

2 **a** 0.6 **b** 6 **c** 7

d –2.5 **e** 3.5 **f** –6

3 $x = 4$

4 **a** 6 **b** 20 **c** 6 or –2

d 2 or –2 **e** 8 or –3 **f** $\pm\sqrt{15}$

5 $x = 60$

6 **a** 3 or $-\frac{3}{2}$ **b** 1 or–2 **c** 2 or 5 **d** 3 or $\frac{1}{12}$

7 **a** $4 + x^2 = 4x$; $x^2 - 4x + 4 = 0$; $(x-2)^2 = 0$ and $x = 2$ is the only solution

b $x = 1$ or 4

c $4 + x^2 = 8x$; $x^2 - 8x + 4 = 0$; using the quadratic formula

$$x = \frac{8 \pm \sqrt{64 - 16}}{2} = \frac{8 \pm \sqrt{48}}{2} = \frac{8 \pm \sqrt{16 \times 3}}{2} = \frac{8 \pm 4\sqrt{3}}{2} = 4 \pm 2\sqrt{3}$$

An alternative method is to complete the square.

13.7 Simultaneous equations

Exercise 13J

1 **a** $x = 5, y = 10$ **b** $x = 18, y = 6$ **c** $x = 12, y = 48$

2 **a** $x = 6, y = 18$ **b** $x = 12.5, y = 2.5$ **c** $x = 0.5, y = 4.5$

3 **a** $x = 13, y = 7$ **b** $x = 9, y = 14$ **c** $x = 10, y = -4$

4 **a** $x = 0.5, y = 4$ **b** $x = 5.5, y = 14.5$ **c** $x = 2, y = 8$

5 Carmen 32, Anish 8

6 11.5 and 25.5

7 8 and –3

8 **a** $x + y = 75$ **b** $y = 2x$ **c** $x = 25, y = 50$

9 **a** $x + y = 300$ **b** $x = y + 60$ or $y = x - 60$

c $x = 180$ and $y = 120$

10 **a** $x = y - 26$ or $y = x + 26$ or $y - x = 26$ **b** $x + y = 50$

c Ahmed is 12 and his mother is 38.

11 **a** $x = y - 0.4$ or $y = x + 0.4$ **b** $x + y = 8.6$ **c** 4.5 m

Exercise 13K

1 **a** $x = 4, y = 1$ **b** $x = 1, y = 4$

c $x = 3, y = 1$ **d** $x = 5, y = -2$

e $x = 7, y = 1$ **f** $x = 5, y = \frac{1}{2}$

g $x = 4\frac{1}{2}, y = 1\frac{1}{2}$ **h** $x = -2, y = 4$

i $x = 2\frac{1}{2}, y = -1\frac{1}{2}$ **j** $x = 2\frac{1}{4}, y = 6\frac{1}{2}$

k $x = 4, y = 3$ **l** $x = 5, y = 3$

2 **a** 3 is the first term. The next term is $3 \times a + b$, which equals 14.

b $14a + b = 47$

c $a = 3, b = 5$

d 146, 443

Exercise 13L

1 **a** $x = 2, y = -3$ **b** $x = 7, y = 3$

c $x = 4, y = 1$ **d** $x = 2, y = 5$

e $x = 4, y = -3$ **f** $x = 1, y = 7$

g $x = 2\frac{1}{2}, y = 1\frac{1}{2}$ **h** $x = -1, y = 2\frac{1}{2}$

i $x = 6, y = 3$ **j** $x = \frac{1}{2}, y = -\frac{3}{4}$

k $x = -1, y = 5$ **l** $x = 1\frac{1}{2}, y = \frac{3}{4}$

2 **a** They are the same equation. Divide the first by 2 and it is the second, so they have an infinite number of solutions.

b Double the second equation to get $6x + 2y = 14$ and subtract to get $9 = 14$. The left-hand sides are the same if the second is doubled so they cannot have different values.

Exercise 13M

1 **a** $x = 5, y = 1$ **b** $x = 3, y = 8$

c $x = 9, y = 1$ **d** $x = 7, y = 3$

e $x = 4, y = 2$ **f** $x = 6, y = 5$

g $x = 3, y = -2$ **h** $x = 2, y = \frac{1}{2}$

i $x = -2, y = -3$ **j** $x = -1, y = 2\frac{1}{2}$

k $x = 2\frac{1}{2}, y = -½$ **l** $x = -1\frac{1}{2}, y = 4\frac{1}{2}$

m $x = -\frac{1}{2}, y = -6\frac{1}{2}$ **n** $x = 3\frac{1}{2}, y = 1\frac{1}{2}$

o $x = -2\frac{1}{2}, y = -3\frac{1}{2}$

2 (1, –2) is the solution to equations A and C; (–1, 3) is the solution to equations A and D; (2, 1) is the solution to B and C; (3, –3) is the solution to B and D.

3 Intersection points are (0, 6), (1, 3) and (2, 4). Area is 2 cm^2

4 Intersection points are (0, 3), (6, 0) and (4, –1). Area is 6 cm^2

13.8 Linear and non-linear simultaneous equations

Exercise 13N

1 **a** (5, –1) **b** (4, 1) **c** (8, –1)

2 **a** (1, 2) and (–2, –1) **b** (–4, 1) and (–2, 2)

3 **a** (3, 4) and (4, 3) **b** (0, 3) and (–3, 0) **c** (3, 2) and (–2, 3)

4 **a** (2, 5) and (–2, –3) **b** (–1, –2) and (4, 3) **c** (3, 3) and (1, –1)

5 **a** (–3, –3), (1, 1) **b** (3, –2), (–2, 3) **c** (–2, –1), (1, 2)

d (2, –1), (3, 1) **e** (–2, 1), (3, 6) **f** (1, –4), (4, 2)

g (4, 5), (–5, –4)

6 **a** $x + y = 12$; $x^2 + y^2 = 90$ **b** Either 391290 or 931290

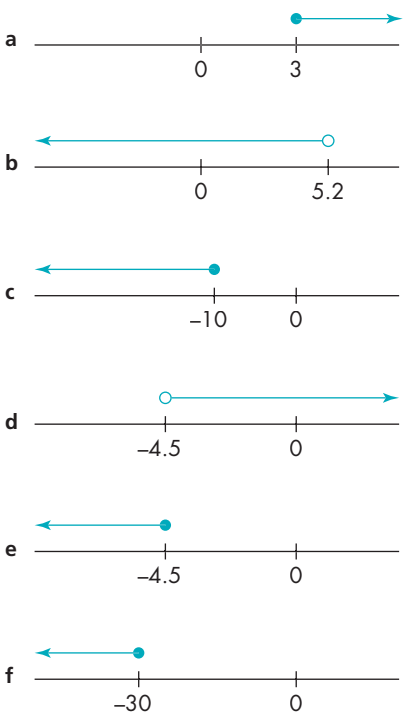
7 12 years old

8 **a** $x^2 + y^2 = 85$ and $(x + y)^2 = 121$ **b** 2 and 9

13.9 Representing inequalities

Exercise 13O

1



2 **a** $x \geqslant -2$

b $x < 7.4$

c $x < -6.5$

d $x \geqslant 1.75$

3 1, 2, 3, 4, 5

4 –3, –2, –1, 0, 1, 2, 3

5 **a**

b

c

d

e

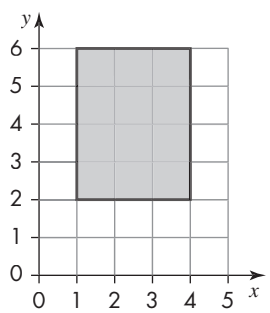
f

6 **a** $0 < x < 5$ **b** $-3 \leqslant x \leqslant 8.5$
c $-10 < x \leqslant -4$
d $-22 \leqslant x \leqslant 3.7$

7 –3, –2, –1, 0, 1, 2

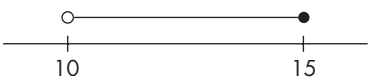
8 **a** $1 < y < 3$

b



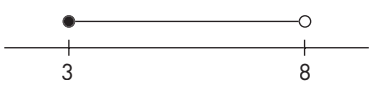
9 **a** $10 < x \leqslant 15$

b



10 **a** $3 \leqslant x < 8$

b



13.10 Solving inequalities

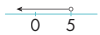
Exercise 13P

1 **a** $x < 3$

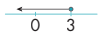
b $t > -2$

c $p \geqslant -10$

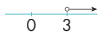
d $x < 5$



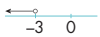
e $y \leqslant 3$



f $t > 3$



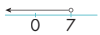
g $x < 6$



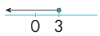
h $y \leqslant -15$

i $t \geqslant 18$

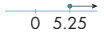
j $x < 7$



k $x \leqslant 3$



l $t \geqslant 5.25$



2 **a** 8 **b** 6 **c** 16
d 3 **e** 7

3 **a** 11 **b** 16 **c** 16

4 $2x + 3 < 20$, $x < 8.50$, so the most each could cost is \$8.49

5 **a** Because 3 + 4 = 7, which is less than the third side of length 8
b $x + x + 2 > 10$, $2x + 2 > 10$, $2x > 8$, $x > 4$, so smallest value of x is 5

6 **a** $x = 6$ and $x < 3$ scores –1 (nothing in common), $x < 3$ and $x > 0$ scores 1 (1 in common for example), $x > 0$ and $x = 2$ scores 1 (2 in common), $x = 2$ and $x \geqslant 4$ scores –1 (nothing in common), so we get $-1 + 1 + 1 - 1 = 0$
b $x > 0$ and $x = 6$ scores +1 (6 in common), $x = 6$ and $x \geqslant 4$ scores +1 (6 in common), $x \geqslant 4$ and $x = 2$ scores –1 (nothing in common), $x = 2$ and $x \leqslant 3$ scores +1 (2 in common), $+1 + 1 - 1 + 1 = 2$
c Any acceptable combination, e.g. $x = 2$, $x \leqslant 3$, $x > 0$, $x \geqslant 4$, $x = 6$

7 **a** $x \geqslant -6$ **b** $t \leqslant \frac{8}{3}$
c $y \leqslant 4$ **d** $x \geqslant -2$
e $w \leqslant 5.5$ **f** $x \leqslant \frac{14}{5}$

8 **a** $x \leqslant 2$ **b** $x > 38$
c $x < 6\frac{1}{2}$ **d** $x \geqslant 7$
e $t > 15$ **f** $y \leqslant \frac{7}{5}$

9 **a** $3 < x < 9$ **b** $-2.5 \leqslant x \leqslant 1.5$ **c** $2 < x \leqslant 5$
d $-30 \leqslant x < 0$ **e** $-0.75 < x < 0.5$ **f** $-15 \leqslant x \leqslant 18$

10 **a** $-3 \leqslant 5x - 3 \leqslant 47$ **b** $-15 \leqslant 5(x - 3) \leqslant 35$

11 **a**

b 4 and 5

12 **a** 4 **b** 99 **c** 11 **d** 11 **e** 6

13 **a** 0, 10 – 10 **b** $x < 16$

14 **a** $x < 9$ **b** $x \geqslant 11$ **c** $x \geqslant 3$

15 **a** $x \geqslant 7.5$ **b** $x \leqslant -2$ **c** $x < 6$
d $x > 1.5$ **e** $x \geqslant -5$ **f** $x < 0.5$

Answers to Chapter 14

14.1 Conversion graphs

Exercise 14A

1 **a** **i** $8\frac{1}{4}$ kg **ii** $2\frac{1}{4}$ kg **iii** 9 lb **iv** 22 lb
b 2.2 lb
c Read off the value for 12 lb (5.4 kg) and multiply this by 4 (21.6 kg)

2 **a** **i** 10 cm **ii** 23 cm **iii** 2 in **iv** $8\frac{3}{4}$ in
b $2\frac{1}{2}$ cm
c Read off the value for 9 in (23 cm) and multiply this by 2 (46 cm)

3 **a** **i** \$320 **ii** \$100 **iii** £45 **iv** £78
b \$3.20
c It would become more steep.

4 **a** **i** \$120 **ii** \$82
b **i** 32 **ii** 48

5 **a** **i** \$100 **ii** \$325
b **i** 500 **ii** 250

6 **a** **i** \$70 **ii** \$29
b **i** \$85 **ii** \$38

7 **a** **i** 95 °F **ii** 68 °F **iii** 10 °C **iv** 32 °C
b 32°F

8 **a** Check student's graph **b** \$50

9 **a** Student's own graph **b** about 48 kilometres
c about 16 miles

10 **a** Student's own graph **b** about 9 centimetres
c about 4 hours

11 **a** Student's own graph **b** about 23 minutes

14.2 Travel graphs

Exercise 14B

1 **a** **i** 2 h **ii** 3 h **iii** 5 h
b **i** 40 km/h **ii** 120 km/h **iii** 40 km/h
c 5.30 am

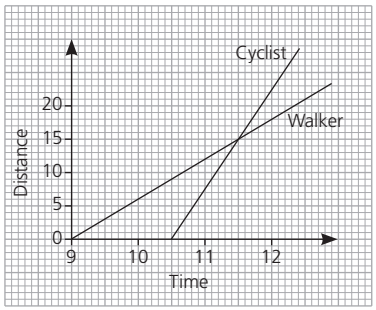
2 **a** **i** 125 km **ii** about 25 km/h
b **i** Between 2 pm and 3 pm **ii** About 12 km/h

3 **a** 30 km **b** 40 km **c** 100 km/h

4 **a** **i** 263 m/min (3 sf)
ii 15.8 km/h (3 sf)
b 500 m/min
c Yuto by 1 minute

5 **a** Patrick ran quickly at first, then had a slow middle section but he won the race with a final sprint. Araf ran steadily all the way and came second. Sean set off the slowest, speeded up towards the end but still came third.
b **i** 1.67 m/s **ii** 6 km/h

6 **a**



b At 1130

7 **a** **i** Because it stopped several times
ii Ravinder
b Ravinder at 1558, Sue at 1620, Michael at 1635
c **i** 24 km/h
ii 20.6 km/h
iii 5

8 **a** 50 metres **b** student's graph **c** 1 metre/second

9 **a** student's graph **b** 80 km/hour

10 **a** 1300 **b** 15 km **c** student's graph
d For the three stages, 5 km/hour, 4 km/hour and 2 km/hour. For the whole trip 3.75 km/hour

14.3 Speed–time graphs

Exercise 14C

1 **a** 20 m/s **b** 0.5 m/s^2
c 1 m/s^2 **d** 600 metres
e 10 m/s

2 **a** 0.6 m/s^2 **b** 750 m

3 **a** 0.2 m/s^2 **b** 0.1 m/s^2
c 75 metres **d** 2.5 m/s

4 **a** 1 m/s^2 **b** $\frac{2}{3}$ m/s^2
c 6 kilometres (or 6000 metres)
d 30 m/s

5 **a** student's graph **b** 0.8 m/s^2 **c** 80 metres

6 **a** 26 m/s **b** student's graph **c** 144 metres

7 **a** $1\frac{1}{3}$ m/s^2
b They are together. They have both travelled 450 metres

8 **a** 2 m/s^2
b **i** after 20 seconds
ii 100 metres
c 1150 metres

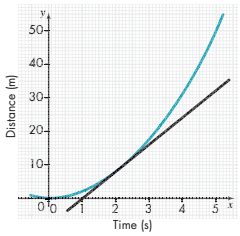
9 **a** 15 seconds **b** $1\frac{1}{3}$ m/s^2

10 **a** 6 m/s **b** student's own graph
c 15 metres

14.4 Curved graphs

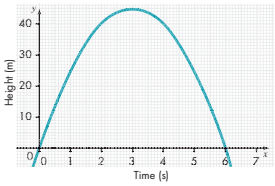
Exercise 14D

1 **a** and **b**



c 8 m/s **d** 16 m/s

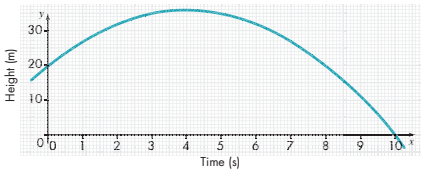
2 **a**



b 10 m/s **c** 30 m/s **d** 0 m/s

e 20 m/s downwards

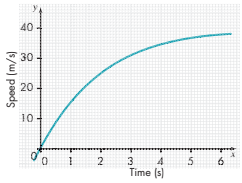
3 **a**



b 4 m/s **c** 6 m/s downwards **d** after 4 s

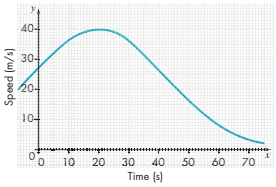
e 12 m/s

4 **a**



b about 12 m/s^2 **c** about 7.4 m/s^2 **d** about 2.7 m/s^2

5 **a**

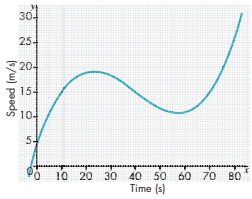


b about 0.73 m/s^2

c after 20 seconds

d about 0.65

6 **a**



b about 0.72 m/s^2 **c** about 0.36 m/s^2

d about 0.72 m/s^2 **e** after about 23 s and 57 s

Answers to Chapter 15

15.1 Drawing straight-line graphs

Exercise 15A

1 Extreme points are (0, 4), (5, 19)

2 Extreme points are (0, –5), (5, 5)

3 Extreme points are (0, –3), (10, 2)

4 Extreme points are (–3, –4), (3, 14)

5 Extreme points are (–6, 2), (6, 6)

6 **a** Extreme points are (0, –2), (5, 13) and (0, 1), (5, 11)
b (3, 7)

7 **a** Extreme points are (0, –5), (5, 15) and (0, 3), (5, 13)
b (4, 11)

8 **a** Extreme points are (0, –1), (12, 3) and (0, –2), (12, 4)
b (6, 1)

9 **a** Extreme points are (0, 1), (4, 13) and (0, –2), (4, 10)
b Do not cross because they are parallel

10 **a** Values of y: 5, 4, 3, 2, 1, 0. Extreme points are (0, 5), (5, 0)
b Extreme points are (0, 7), (7, 0)

11 **a** yes **b** no **c** yes **d** no **e** yes **f** no

12 **a** 6 **b** 3.5 **c** 2

13 **a** 20 **b** –10

15.2 The equation $y = mx + c$

Exercise 15B

1 **a** 3 **b** 2 **c** $\frac{1}{2}$
d 3 **e** $\frac{1}{3}$

2 **a** $y = 2x - 2$ **b** $y = x + 1$ **c** $y = 2x - 3$
d $2y = x + 6$ **e** $y = x$ **f** $y = 2x$

3 **a** $y = 2x + 1$, $y = -2x + 1$ **b** $5y = 2x - 5$, $5y = -2x - 5$
c $y = x + 1$, $y = -x + 1$

4 **a** $y = -2x + 1$ **b** $2y = -x$ **c** $y = -x + 1$
d $5y = -2x - 5$ **e** $y = -\frac{3}{2}x - 3$ or $2y = -3x - 6$

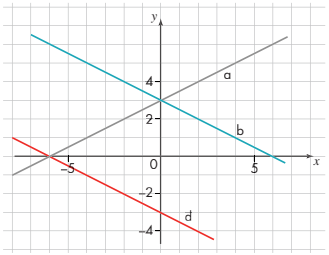
5 **a** 3 **b** (0,3)

6 **a** 4 **b** 4

7 The first and last are parallel because they both have a gradient of 4. The middle one has a gradient of 3.

8 y = 0.6x

9 **a**, **b** and **d**



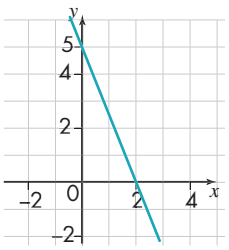
c $y = -0.5x + 3$ **e** $y = -0.5x - 3$

15.3 More about straight-line graphs

Exercise 15C

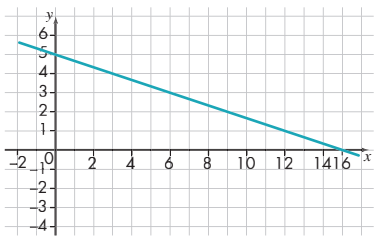
1 **a** $y = -2.5x + 5$ **b** –2.5 and (0, 5)

c



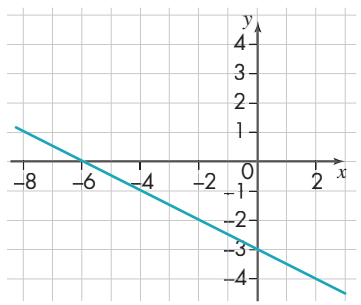
2 **a** $y = -\frac{1}{3}x + 5$ **b** $-\frac{1}{3}$ and (0, 5)

c



3 **a** $y = -0.5x - 3$ **b** –0.5 and (0, –3)

c



4 **a** $y = -x - 20$ **b** $y = 3x + 15$ **c** $y = -0.7x + 3$
d $y = 0.2x - 8$ **e** $y = 1.5x - 1$ **f** $y = -0.5x + 6$

5 **a** –1 and (0, –20) **b** 3 and (0, 15) **c** –0.7 and (0, 3)
d 0.2 and (0, –8) **e** 1.5 and (0, –1) **f** –0.5 and (0, 6)

6 **a** –1 **b** $\frac{1}{3}$ **c** 6
d –0.5 **e** 2 **f** 5.6

7 **a** C **b** A **c** D **d** B

8 line d, $3x - 2y = 12$, all the rest are the same line.

9 **a** (9, 0) and (0, 15)
b (20, 0) and (0, 10)
c (0, –10) and (–5, 0)

15.4 Solving equations graphically

Exercise 15D

1 **a** 1.8 **b** –0.4 **c** 2.7

2 **a** –1.6 **b** 3.8 or 3.9 **c** –3.8

3 **a** 2.3 **b** 1.6 **c** 0.9

4 **a** 23 **b** –29 **c** 71

5 **a** 6.9 **b** 2.9 **c** 4.2 or 4.3

6 **a** –0.8 **b** 2.6 **c** 0.3

15.5 Parallel lines

Exercise 15E

1 **a** $2 \times 3 + 6 = 12$ **b** 2 **c** student's graph
d $y = 2x$ **e** $y = 2x + 3$

2 **a** (0, –1) and (4, 0) **b** $y = \frac{1}{4}x + 3$

3 **a** –2 **b** (4,0) **c** student's graph
d $y = -2x$ **e** $y = -2x + 14$

4 **a** 1 **b** $y = 5x - 9$

5 $y = \frac{2}{3}x - 3$

6 **a** $y = 2x - 4$ **b** $y = 2x + 8$

7 **a** $4 + 2 \times 1 = 6$ **b** $y = -2x + 6$
c –2 **d** $y = -2x$

15.6 Points and lines

Exercise 15F

1 **a** 3 **b** $\frac{1}{2}$ **c** 4 **d** –1 **e** $-\frac{1}{2}$ **f** $\frac{2}{3}$

2 **a** $y = 2x - 3$ **b** $y = \frac{1}{2}x + 4$ **c** $y = 4x - 2$ **d** $y = -3x + 8$

3 **a** (5, 3) **b** (4, 5) **c** (3, 2)
d (3, 3) **e** (1, 3.5) **f** (–0.5, 0)

4 **a** student's graph **b** $y = 0.5x + 6.5$ **c** (–1,3)
d $y = -x + 8$

5 **a** 5 **b** 13 **c** 10 **d** 17

6 Show that the distance from each point to (2,1) is 5.

7 AB = $\sqrt{32}$, AC = $\sqrt{80}$, BC = $\sqrt{80}$, so two of the sides are the same length.

15.7 Perpendicular lines

Exercise 15G

1 **a** –2 **b** $-\frac{1}{4}$ **c** $\frac{3}{2}$

2 The gradients are 5 and $-\frac{1}{5}$; $5 \times -\frac{1}{5} = -1$

3 **a** $-\frac{1}{3}$ **b** –2 **c** 2.5 **d** $-\frac{2}{15}$

4 **a** 1 **b** 2 **c** $\frac{4}{3}$ **d** –6

5 **a** $y = -\frac{1}{5}x$ **b** $y = -\frac{1}{5}x + 10$ **c** $y = -\frac{1}{5}x + 2$

6 $y = \frac{3}{5}x + 2$

7 $y = -\frac{1}{4}x + \frac{1}{2}$

8 $y = -5x + 11$

9 $6x + 3y = 7$ is the odd one out

10 $y = 2x + 6$

Answers to Chapter 16

16.1 Quadratic graphs

Exercise 16A

1 **a** x: –3, –2, –1, 0, 1, 2, 3
y: 11, 6, 3, 2, 3, 6, 11
b student's graph

2 **a** x: –3, –2, –1, 0, 1, 2, 3, 4, 5W
x^2: 9, 4, 1, 0, 1, 4, 9, 16, 25
$-3x$: 9, 6, 3, 0, –3, –6, –9, –12, –15
y: 18, 10, 4, 0, –2, –2, 0, 4, 10
b 1.8 **c** (1.5, –2.25)
d $x = 1.5$ **e** $x = 4.2$ or –1.2

3 **a** y: 7, 0, –5, –8, –9, –8, –5, 0, 7 **b** $x = 4$ or –2
c The graph should give a value of about –8.75
d The graph should give values of about 4.5 and –2.5

4 **a** y: 10, 4, 0, –2, –2, 0, 4, 10
b (2.5, –2.25)
c $x = 2.5$
d The graph should give a value of about 6.75
e x = 1 or 4
f The graph should give values of about 0.2 and 4.8

5 **a** x: –4, –3, –2, –1, 0, 1, 2
y: 7, 2, –1, –2, –1, 2, 7
b 0.4, –2.4
c 1.6, –3.6
d $x = –2.7$ or 0.7

6 **a** x: –4, –3, –2, –1, 0, 1, 2, 3, 4
y: –4, 3, 8, 11, 12, 11, 8, 3, –4
b 9.75
c ± 3.5
d 2.2 and –2.2

7 **a**

x	–5	–4	–3	–2	–1	0	1	2
x^2	25	16	9	4	1	0	1	4
$+4x$	–20	–16	–12	–8	–4	0	4	8
y	5	0	–3	–4	–3	0	5	12

b $x = –4$ and 0
c –3.8
d –4, 0

8 **a** x: –1, 0, 1, 2, 3, 4, 5, 6, 7
y: 10, 3, –2, –5, –6, –5, –2, 3, 10
b $x = 0.6$ or 5.5
c –5.8
d –0.3, 6.5

9 **a** y values: –6, 0, 4, 6, 6, 4, 0, –6
b student's graph
c (2.5, 6.25)
d $x = 2.5$
e $x = 4.6$ and 0.4

16.2 Solving equations graphically

Exercise 16B

1 **a**

x	–3	–2	–1	0	1	2	3
x^2	9	4	1	0	1	4	9
$x + 4$	1	2	3	4	5	6	7

b Student's graphs
c $x = 2.6$ and –1.6 (to 1 d.p.)

2 **a**

x	–2	–1	0	1	2	3	4	5
x^2	4	1	0	1	4	9	16	25
$5x – 2$	–12	–7	–2	3	8	13	18	23

b Student's graphs
c $x = 0.4$ and 4.6 (to 1 d.p.)

3 **a**

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$10 – x$	14	13	12	11	10	9	8	7	6

b Student's graph; $x = 2.7$ and –3.7 (to 1 d.p.)

4 **a**

x	–5	–4	–3	–2	–1	0	1	2	3	4	5
x^2	25	16	9	4	1	0	1	4	9	16	25
$3x + 5$	–10	–7	–4	–1	2	5	8	11	14	17	20

b $x = 4.2$ and –1.2 (to 1 d.p.)

5 **a**

x	–4	–3	–2	–1	0	1	2	3	4
$5 – x^2$	–11	–4	1	4	5	4	1	–4	–11
$2x$	–8	–6	–4	–2	0	2	4	6	8

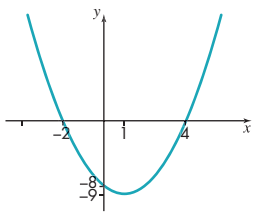
b $x = 1.4$ and –3.4 (to 1 d.p.)

6 $x = 1.6$ and –2.6 (to 1 d.p.)

16.3 Turning points on a quadratic graph

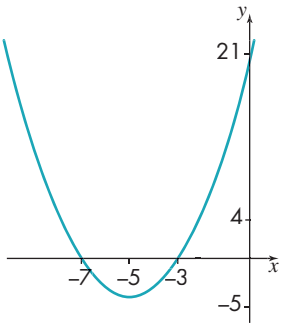
Exercise 16C

1 **a** $x^2 – 2x – 8 = (x – 1)^2 – 9$ **b** (1, –9) **c** $x = 4$ or –2
d



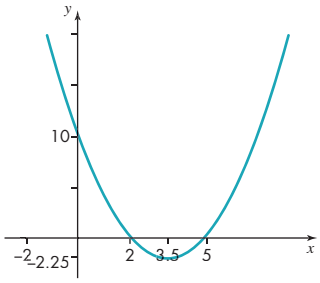
2 **a** $x^2 + 10x + 21 = (x + 5)^2 - 4$ **b** (–5, –4) **c** $x = -3$ or -7

d



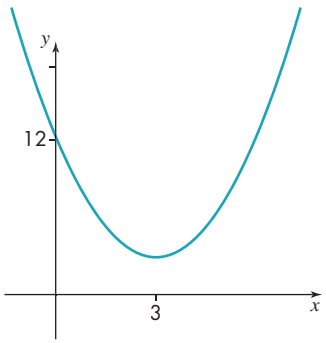
3 **a** $x^2 - 7x + 10 = (x - 3.5)^2 - 2.25$ **b** (3.5, –2.25)

c $x = 2$ or 5 **d**



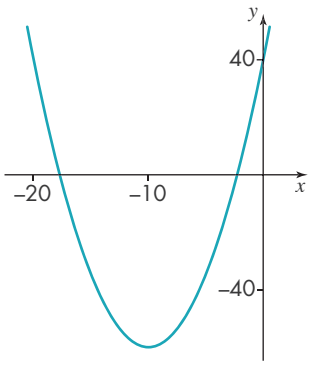
4 **a** (0, 12) **b** (3, 3)

c



d The graph does not cross the x-axis so there is no value of x for which $x^2 - 6x + 12 = 0$

5 **a** (–10, 60) **b**



6 –56.5

7 $b = -10$ and $c = 14$

16.4 Reciprocal graphs

Exercise 16D

1 **a** y values: 10, 5, 4, 2.5, 2, 1.33, 1, 0.67, 0.5
b 0.8 **c** 0.3 **d** –1.6

2 **a** y values: 25, 12.5, 10, 5, 2.5, 1, 0.5, 0.33, 0.25
b Student's graph **c** –0.5 and –9.5

3 Student's own graph

4 **a** y values 20, 10, 5, 4, 2.5, 2, 1
b Student's graph **c** Student's graph **d** $x = 6.5$ or -1.5

16.5 More graphs

Exercise 16E

1 Student's own graph

2 **a** y values: –7.81, –4, –1.69, –0.5, –0.06, 0, 0.06, 0.5, 1.69, 4, 7.81
b 2.3

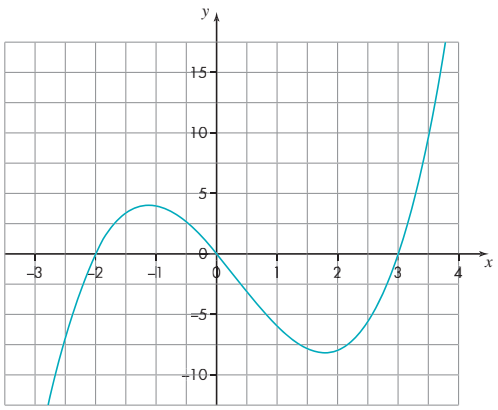
3 **a** y values: –12.63, –5, –0.38, 2, 2.89, 3, 3.13, 4, 6.38, 11, 18.63
b –1.4

4 **a** y values: 1, 4.63, 6, 5.88, 5, 4.13, 4, 5.38, 9
b $x = -1.8$ **c** x = 1.8 **d** (0.8, 3.9) and (–0.8, 6.1)

5 **a**

x	–3	–2	–1	0	1	2	3	4
y	–18	0	4	0	–6	–8	0	24

b

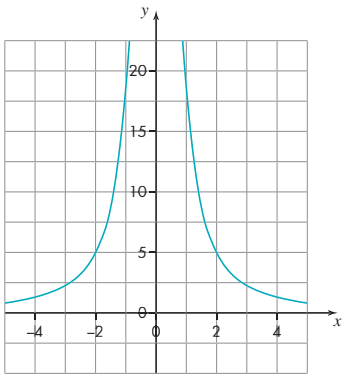


c $x = -2.4$, 0.8 or 2.6 **d** (–1.1, 4.1) and (1.8, –8.2)

6 **a** y values: 20, 5, 2.22, 1.25, 0.8 **b** Student's graph
c $x = 1.6$

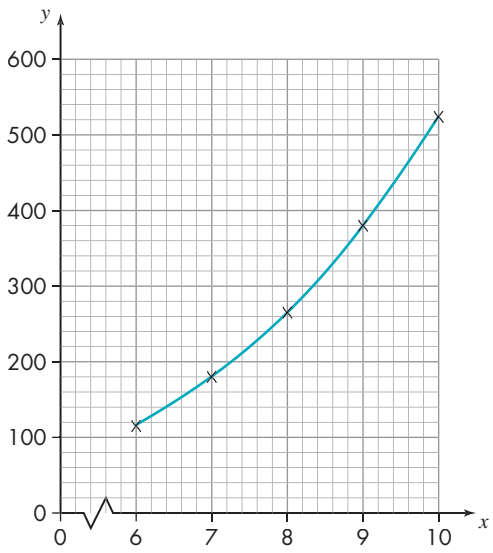
7 **a** y values: 4.25, 1.5, –0.22, –1.46, –2.44

b



c About 5.85

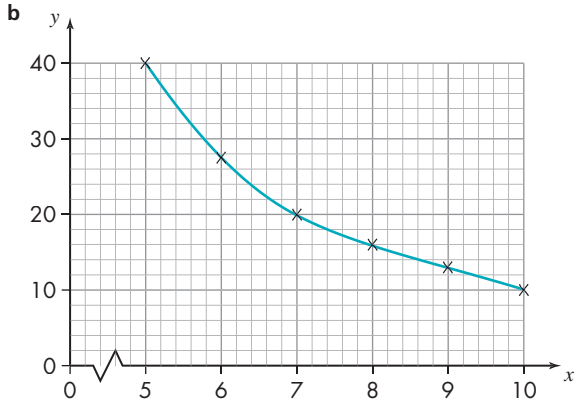
8 **a**



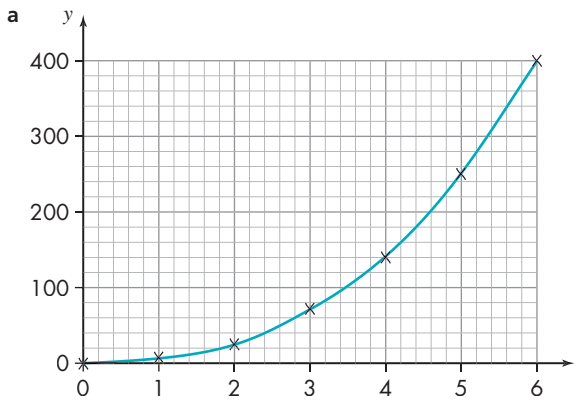
b 8.3 cm

9 **a** $x^2y = 1000 \Rightarrow y = \frac{1000}{x^2}$

b



10 **a**

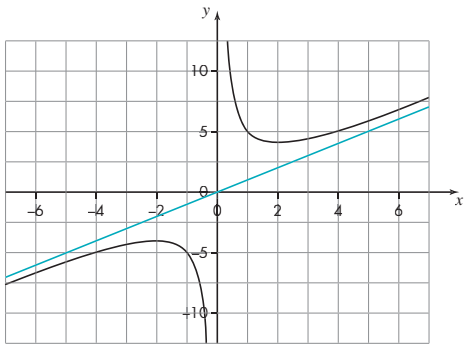


b 4.6 cm, 4.6 cm and 9.6 cm

11 **a**

x	–5	–4	–3	–2	–1	0	1	2	3	4	5
y	–5.8	–5	–4.33	–4	–5	-	5	4	4.33	5	5.8

b and **c**



d The values of y are 20.4 and 20 and these numbers are close together.

16.6 Square root graphs

Exercise 16F

1 **a**

x	0	1	2	3	4	5	6	7	8	9	10
$2+\sqrt{x}$	2	3	3.41	3.73	4	4.24	4.45	4.65	4.83	5	5.16

b Student's graph

c $x = 2.6$

d Student's graph

e $x = 4$

2 **a**

x	0.5	1	2	3	4	5	6	7	8	9	10
$\frac{4}{\sqrt{x}}$	5.66	4	2.83	2.31	2	1.79	1.63	1.51	1.41	1.33	1.26

b Student's graph

c $x = 2.8$

d $x = 2.5$

3 **a**

x	1	2	3	4	5	6
$\sqrt{x}+\frac{10}{x}$	11	6.4	5.1	4.5	4.2	4.1

b Student's graph

c $x = 2.2$

4 **a**

x	1	5	10	15	20	25
$1+\frac{20}{\sqrt{x}}$	21	9.9	7.3	6.2	5.5	5

b Student's graph

c Student's graph

d $x = 13.1$

5 **a** Student's graph

b $x = 3.2$

c Student's graph

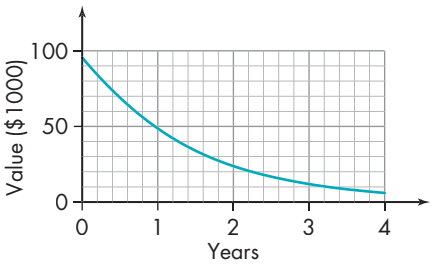
d $x = 1.8$

16.7 Exponential graphs

Exercise 16G

1 **a** The missing values are 48, 24, 12 and 6

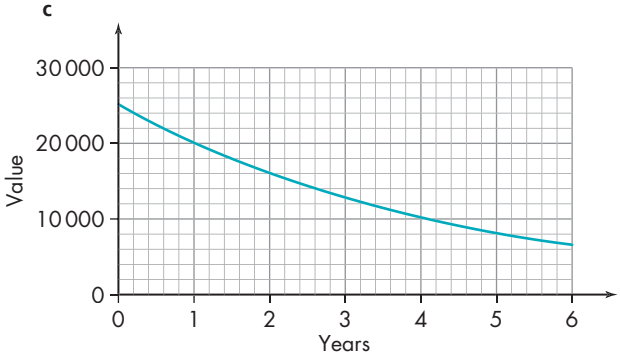
b



2 **a** 25 000 × 0.8 = 20 000

b The missing values are 16 000, 12 800, 10 240, 8192

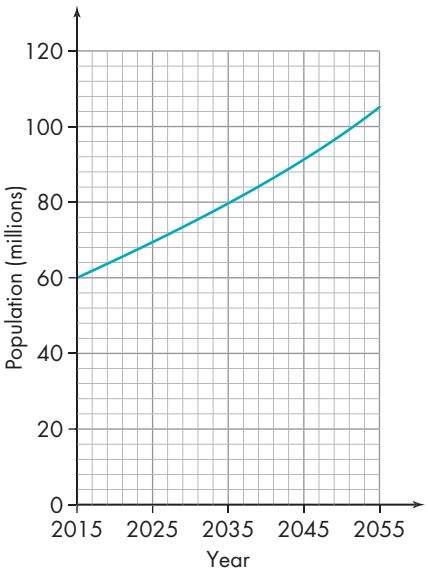
c



d 3.1 years

3 **a** The missing numbers, rounded to one decimal place, are 79.4, 91.3 and 104.9

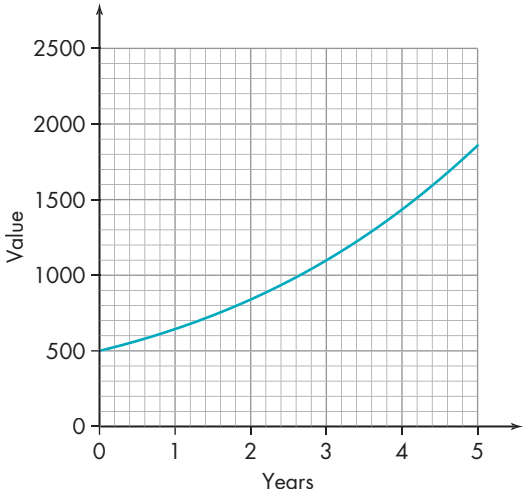
b



c 2051 or 2052

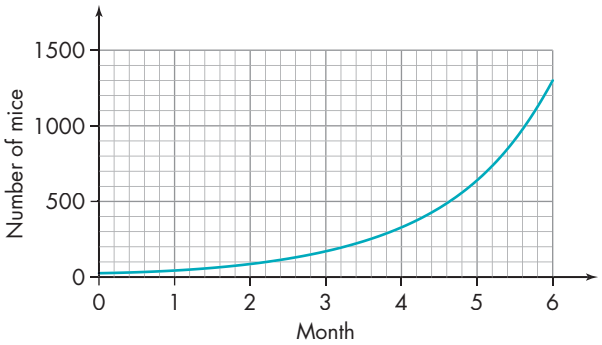
4 **a** the missing values, to the nearest whole number, are 650, 845, 1099, 1428

b



c about 2.6 or 2.7 years

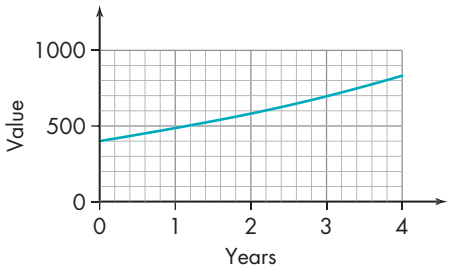
5 **a**



b about 3.3 months

6 **a** 20%

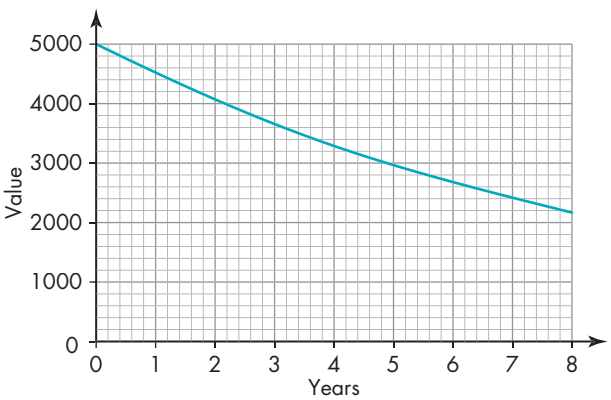
b



c About $630

7 **a** The missing values are 4500, 3645 and 3280.5

b



c About 6.6 years

8 It is not exponential growth because the gradient is decreasing. The gradient increases as time passes in exponential growth.

9 **a** 200 **b** 50% **c** about 2.7 hours

10 **a** The initial value is \$1000. After two years it is \$500. After four years it is \$250. After six years it is \$125. The value halves every two years.
b 29 or 30%

16.8 Estimating gradients

Exercise 16H

Gradients found in this exercise may vary from the answers given due to variations in drawings of the tangents

1 0.67

2 A: 0.5 B: –2

3 **a** student's drawing
b student's drawing
c about 1.8
d (1, 1.5)

4 **a** y values: 0, 0.01, 0.1, 0.34, 0.8, 1.56, 2.7
b student's drawing
c student's drawing
d about 1.2

5 **a** y values: 0.25, 0.5, 1, 2, 4
b student's drawing
c student's drawing
d about 0.7

6 **a** y values: 2.5, 1.67, 1.25, 1, 0.83
b student's drawing
c about –0.3

Answers to Chapter 17

17.1 Patterns in number sequences

Exercise 17A

1 **a** 9, 11, 13: add 2
b 10, 12, 14: add 2
c 80, 160, 320: double
d 81, 243, 729: multiply by 3
e 28, 34, 40: add 6
f 23, 28, 33: add 5
g 20 000, 200 000, 2 000 000: multiply by 10
h 19, 22, 25: add 3
i 114, 105, 96: subtract 9
j 405, 1215, 3645: multiply by 3
k 25, 12.5, 6.25; halve
l 625, 3125, 15 625: multiply by 5

2 **a** 16, 22 **b** 26, 37
c 31, 43 **d** 46, 64
e 121, 169 **f** 782, 3907
g 22 223, 222 223 **h** 11, 13
i 33, 65 **j** 78, 108

3 **a** 48, 96, 192 **b** 33, 39, 45
c 4, 2, 1 **d** 38, 35, 32
e 37, 50, 65 **f** 26, 33, 41
g 14, 16, 17 **h** 25, 22, 19
i 28, 36, 45 **j** 5, 6, 7
k 0.16, 0.032, 0.0064
l 0.0625, 0.031 25, 0.015 625

4 **a** 21, 34: add previous 2 terms
b 49, 64: next square number
c 47, 76: add previous 2 terms
d 216, 343: cube numbers

5 15, 21, 28, 36

6 61, 91, 127

7 29 and 41

8 No, they both increase by the same number (3).

9 10, 45 and 80

17.2 The nth term of a sequence

Exercise 17B

1 **a** 3, 5, 7, 9, 11 **b** 1, 4, 7, 10, 13
c 7, 12, 17, 22, 27 **d** 1, 4, 9, 16, 25
e 4, 7, 12, 19, 28 **f** 18, 16, 14, 12, 10

2 **a** 4, 5, 6, 7, 8 **b** 2, 5, 8, 11, 14
c 3, 8, 13, 18, 23 **d** 0, 3, 8, 15, 24
e 9, 13, 17, 21, 25 **f** 42, 39, 36, 33, 30

3 **a** 94, 88, 82, 76 **b** the 17th term, –2

4 **a** 1, 4, 9, 16 **b** 3, 6, 11, 18 **c** 2, 8, 18, 32
d 3, 15, 35, 63 **e** 199, 196, 191, 184
f 0.25, 1, 2.25, 4

5 **a** 1, 8, 27, 64 **b** 2, 9, 28, 65 **c** –1, 6, 25, 62
d 2, 23, 80, 191 **e** 0.5, 4, 13.5, 32 **f** 108, 101, 82, 45

6 **a** \$305 **b** \$600 **c** 3
d 5 (the amount is \$250)

7 $4n - 2 = 3n + 7$ rearranges as $4n - 3n = 7 + 2$, $n = 9$

Exercise 17C

1 **a** 13, 15, $2n + 1$ **b** 25, 29, $4n + 1$
c 33, 38, $5n + 3$ **d** 32, 38, $6n - 4$
e 20, 23, $3n + 2$ **f** 37, 44, $7n - 5$
g 17, 15; $29 - 2n$ **h** 22, 18; $46 - 4n$
i 17, 20, $3n - 1$ **j** 42, 52, $10n - 8$
k 24, 28, $4n + 4$ **l** 29, 34, $5n - 1$

2 **a** $3n + 1$, 151 **b** $2n + 5$, 105
c $5n - 2$, 248 **d** $4n - 3$, 197
e $8n - 6$, 394 **f** $n + 4$, 54
g $5n + 1$, 251 **h** $8n - 5$, 395
i $3n - 2$, 148 **j** $3n + 18$, 168
k $7n + 5$, 355 **l** $8n - 7$, 393

3 **a** **i** $4n + 1$ **ii** 401
b **i** $2n + 1$ **ii** 201
c **i** $3n + 1$ **ii** 301
d **i** $2n + 6$ **ii** 206
e **i** $4n + 5$ **ii** 405

f **i** $5n + 1$ **ii** 501
g **i** $3n - 3$ **ii** 297
h **i** $6n - 4$ **ii** 596
i **i** $8n - 1$ **ii** 799
j **i** $2n + 23$ **ii** 223

4 **a** $8n + 2$ **b** $8n + 1$ **c** $8n$ **d** \$8

5 **a** n^2 **b** $n^2 + 2$ **c** $2n^2$ **d** $n^2 - 1$

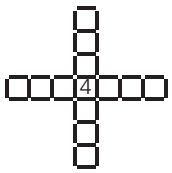
6 **a** n^3 **b** $n^3 + 10$ **c** $0.5n^3$ **d** $10n^3$

7 **a** $n + 5$ **b** $n^2 + 5$ **c** $n^3 + 5$
d $5n + 1$ **e** $5n^2$ **f** $5n^3$

17.3 General rules from patterns

Exercise 17D

1 **a**



b The missing number is 13
c $4n - 3$
d 97
e 50th diagram

2 **a** △▽△▽△
b The bottom line is 3, 5, 7, 9, 11
c $2n + 1$
d 121
e 49th set

3 **a** 18
b the bottom line is 6, 10, 14, 18
c $4n + 2$
d 12

4 **a** **i** 24 **ii** $5n - 1$ **iii** 224
b 25

5 **a** **i** 20 cm **ii** $(3n + 2)$ cm **iii** 152 cm
b 332

6 **a** **i** 10 **ii** $2n + 2$ **iii** 162
b 79.8 km

7 **a** **i** 14 **ii** $3n + 2$ **iii** 41
b 66

8 **a** **i** 5 **ii** n **iii** 18
b Formula gives 3 and 6
c 55

9 **a**

1	**2**	**3**
3	9	27
1	4	13
4	13	40

b the numbers in column 4 (top to bottom) are: 4, 81, 40, 121. Student's explanation of method.

10 **a** Student's drawing – one complete recurring sequence should be added to each one.
b bottom row: 5, 9, 13, 17
c $4n + 1$
d $3n + 1$
e $2n + 1$
f $9n + 3$

17.4 Further sequences

Exercise 17E

1 **a** 432 **b** 1053 **c** 1250 **d** 41.472
e 640 000 **f** 15 **g** 1.6 **h** 32

2 **a** 6, 1296 **b** 16, 128 **c** 15, 405
d 20, 2.5 **e** 54, 16

3 **a** 25×2^n **b** 1.5×2^n **c** 2×3^n **d** 240×0.5^n
e 50×0.9^n **f** 64×1.25^n

4 **a** 13 720, 19 208 **b** 5000×1.4^n

5 **a** $n^2 + 1$ **b** $n^2 + 6$ **c** $n^2 - n$ **d** $3n^2$
e $3n^2 - 2$ **f** $3n^2 + n$

6 **a** 15, 21 **b** $n^2 + n$ **c** $0.5(n^2 + n)$ or $0.5n^2 + 0.5n$
d 210 **e** It is the 50th triangular number because $(50^2 + 50) \div 2 = 1275$

7 **a** $n^3 - 1$ **b** $n^3 + 50$ **c** $n^3 + n$ **d** $n^3 + 3n$ **e** $4n^3$ **f** $4n^3 - n$

8 **a** 20 **b** $(4 \times 5 \times 6) \div 6 = 20$ **c** 7 (using 84 oranges)
d The layers of the tetrahedron are triangular numbers. 20 layers have $(20 \times 21 \times 22) \div 6 = 1540$ oranges.

9 **a** When $n = 1$ the first term is $a + b$ and this is 6.
b $2a + 4b = 16$ **c** $a = 4$, $b = 2$; the nth term is $4n + 2n^2$

Answers to Chapter 18

18.1 Using indices

Exercise 18A

1 **a** 2^4 **b** 3^5 **c** 7^2 **d** 5^3 **e** 10^7
f 6^4 **g** 4^1 **h** 1^7 **i** 0.5^4 **j** 100^3

2 **a** $3 \times 3 \times 3 \times 3$
b $9 \times 9 \times 9$
c 6×6
d $10 \times 10 \times 10 \times 10 \times 10$
e $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
f 8
g $0.1 \times 0.1 \times 0.1$
h 2.5×2.5
i $0.7 \times 0.7 \times 0.7$
j 1000×1000

3 **a** 16 **b** 243
c 49 **d** 125
e 10 000 000 **f** 1296
g 4 **h** 1
i 0.0625 **j** 1 000 000

4 **a** 81 **b** 729
c 36 **d** 100 000
e 1024 **f** 8
g 0.001 **h** 6.25
i 0.343 **j** 1 000 000

5 125 m^3

6 **b** 10^2 **c** 2^3 **d** 5^2

7 **a** 1 **b** 4 **c** 1 **d** 1 **e** 1

8 Any power of 1 is equal to 1.

9 10^6

10 10^6

11 **a** 1 **b** –1 **c** 1 **d** 1 **e** –1

12 **a** 1 **b** –1 **c** –1 **d** 1 **e** 1

13 2^{24}, 4^{12}, 8^8, 16^6

18.2 Negative indices

Exercise 18B

1 **a** $\frac{1}{5^3}$ **b** $\frac{1}{6}$ **c** $\frac{1}{10^5}$ **d** $\frac{1}{3^2}$
e $\frac{1}{8^2}$ **f** $\frac{1}{9}$ **g** $\frac{1}{w^2}$ **h** $\frac{1}{t}$
i $\frac{1}{x^m}$ **j** $\frac{4}{m^3}$

2 **a** 3^{-2} **b** 5^{-1} **c** 10^{-3} **d** m^{-1} **e** t^{-n}

3 **a** **i** 2^4 **ii** 2^{-1} **iii** 2^{-4} **iv** -2^3
b **i** 10^3 **ii** 10^{-1} **iii** 10^{-2} **iv** 10^6
c **i** 5^3 **ii** 5^{-1} **iii** 5^{-2} **iv** 5^{-4}
d **i** 3^2 **ii** 3^{-3} **iii** 3^{-4} **iv** -3^5

4 **a** $\frac{5}{x^3}$ **b** $\frac{6}{t}$ **c** $\frac{7}{m^2}$ **d** $\frac{4}{q^4}$
e $\frac{10}{y^5}$ **f** $\frac{1}{2x^3}$ **g** $\frac{1}{2m}$ **h** $\frac{3}{4t^4}$
i $\frac{4}{5y^3}$ **j** $\frac{7}{8x^5}$

5 **a** $7x^{-3}$ **b** $10p^{-1}$ **c** $5t^{-2}$ **d** $8m^{-5}$ **e** $3y^{-1}$

6 **a** **i** 25 **ii** $\frac{1}{125}$ **iii** $\frac{4}{5}$
b **i** 64 **ii** $\frac{1}{16}$ **iii** $\frac{5}{256}$
c **i** 8 **ii** $\frac{1}{32}$ **iii** $\frac{9}{2}$ or $4\frac{1}{2}$
d **i** 1 000 000 **ii** $\frac{1}{1000}$ **iii** $\frac{1}{4}$

7 24 (32 – 8)

8 $x = 8$ and $y = 4$ (or $x = y = 1$)

9 $\frac{1}{2097152}$

10 **a** x^{-5}, x^0, x^5 **b** x^5, x^0, x^{-5} **c** x^5, x^{-5}, x^0

11 **a** $\frac{M}{3}$ **b** 3M **c** 27M

18.3 Multiplying and dividing with indices

Exercise 18C

1 **a** 5^4 **b** 5^3 **c** 5^2 **d** 5^3 **e** 5^{-5}

2 **a** 6^3 **b** 6^0 **c** 6^6 **d** 6^{-7} **e** 6^2

3 **a** a^3 **b** a^5 **c** a^7 **d** a^4 **e** a^2 **f** a^1

4 **a** Any two values such that $x + y = 10$
b Any two values such that $x - y = 10$

5 **a** 4^6 **b** 4^{15} **c** 4^6
d 4^{-6} **e** 4^6 **f** 4^0

6 **a** $6a^5$ **b** $9a^2$ **c** $8a^6$
d $-6a^4$ **e** $8a^8$ **f** $-10a^{-3}$

7 **a** $3a$ **b** $4a^3$ **c** $3a^4$
d $6a^{-1}$ **e** $4a^7$ **f** $5a^{-4}$

8 **a** $8a^5b^4$ **b** $10a^3b$ **c** $30a^{-2}b^{-2}$
d $2ab^3$ **e** $8a^{-5}b^7$

9 **a** $3a^3b^2$ **b** $3a^2c^4$ **c** $8a^2b^2c^3$

10 **a** Possible answer: $6x^2 \times 2y^5$ and $3xy \times 4xy^4$
b Possible answer: $24x^2y^7 \div 2y^2$ and $12x^6y^8 \div x^4y^3$

11 12 ($a = 2$, $b = 1$, $c = 3$)

12 **a** A^2 **b** A^{-1} **c** $A^{\frac{1}{2}}$ or $\sqrt{A}$ **d** $A^{\frac{1}{3}}$ or $\sqrt[3]{A}$

13 **a** $x^{2n+1} = x^{2n}x = (x^n)^2 \times x = xy^2$ **b** $\frac{y^2}{x}$

18.4 Fractional indices

Exercise 18D

1 **a** 5 **b** 10 **c** 8 **d** 9 **e** 25
f 3 **g** 4 **h** 10 **i** 5 **j** 8
k 12 **l** 20 **m** 5 **n** 3 **o** 10
p 3 **q** 2 **r** 2 **s** 6 **t** 6
u $\frac{1}{4}$ **v** $\frac{1}{2}$ **w** $\frac{1}{3}$ **x** $\frac{1}{5}$ **y** $\frac{1}{10}$

2 **a** $\frac{5}{6}$ **b** $1\frac{2}{3}$ **c** $\frac{8}{9}$ **d** $1\frac{4}{5}$ **e** $\frac{5}{8}$
f $\frac{3}{5}$ **g** $\frac{1}{4}$ **h** $2\frac{1}{2}$ **i** $\frac{4}{5}$ **j** $1\frac{1}{7}$

3 $(x^{\frac{1}{n}})^n = x^{\frac{1}{n} \times n} = x^1 = x$, but $(\sqrt[n]{x})^n = \sqrt[n]{x} \times \sqrt[n]{x} \ldots n$ times $= x$, so $x^{\frac{1}{n}} = \sqrt[n]{x}$

4 $64^{-\frac{1}{2}} = \frac{1}{8}$, others are both $\frac{1}{2}$

5 Possible answer: The negative power gives the reciprocal, so $27^{-\frac{1}{3}} = \frac{1}{27^{\frac{1}{3}}}$. The power one-third means cube root, so you need the cube root of 27 which is 3, so $27^{\frac{1}{3}} = 3$ and $\frac{1}{27^{\frac{1}{3}}} = \frac{1}{3}$

6 Possible answer: $x = 1$ and $y = 1$, $x = 8$ and $y = \frac{1}{64}$.

7 **a** 3 **b** $\frac{1}{3}$ **c** 0 **d** $\frac{1}{2}$
e $\frac{1}{2}$ **f** $\frac{1}{4}$ **g** $\frac{1}{4}$ **h** $\frac{1}{3}$
i $\frac{1}{3}$ **j** $\frac{1}{2}$ **k** $\frac{1}{3}$ **l** $\frac{1}{7}$

Exercise 18E

1 **a** 16 **b** 25 **c** 216 **d** 81

2 **a** $t^{\frac{2}{3}}$ **b** $m^{\frac{3}{4}}$ **c** $k^{\frac{2}{5}}$ **d** $x^{\frac{3}{2}}$

3 **a** 4 **b** 9 **c** 64 **d** 3125

4 **a** $\frac{1}{5}$ **b** $\frac{1}{6}$ **c** $\frac{1}{2}$ **d** $\frac{1}{3}$
e $\frac{1}{4}$ **f** $\frac{1}{2}$ **g** $\frac{1}{2}$ **h** $\frac{1}{3}$

5 **a** $\frac{1}{125}$ **b** $\frac{1}{216}$ **c** $\frac{1}{8}$ **d** $\frac{1}{27}$
e $\frac{1}{256}$ **f** $\frac{1}{4}$ **g** $\frac{1}{4}$ **h** $\frac{1}{9}$

6 **a** $\frac{1}{100\,000}$ **b** $\frac{1}{12}$ **c** $\frac{1}{25}$ **d** $\frac{1}{27}$
e $\frac{1}{32}$ **f** $\frac{1}{32}$ **g** $\frac{1}{81}$ **h** $\frac{1}{13}$

7 $8^{-\frac{2}{3}} = \frac{1}{4}$, others are both $\frac{1}{8}$

8 Possible answer: The negative power gives the reciprocal, so the power one-third means cube root, so we need the cube root of 27 which is 3 and the power 2 means square, so $3^2 = 9$, so $27^{\frac{2}{3}} = 9$ and $\frac{1}{27^{\frac{2}{3}}} = \frac{1}{9}$

9 **a** $\frac{27}{8}$ **b** $\frac{9}{25}$ **c** $\frac{1024}{243}$ **d** $\frac{8}{343}$
e $\frac{16}{9}$ **f** $\frac{8}{27}$ **g** $\frac{625}{256}$ **h** $\frac{32}{243}$

10 **a** $\frac{25}{9}$ **b** $\frac{27}{64}$ **c** $\frac{125}{729}$ **d** $\frac{243}{32}$
e $\frac{8}{27}$ **f** $\frac{243}{32}$ **g** $\frac{9}{4}$ **h** $\frac{125}{343}$
i $\frac{16}{25}$ **j** $\frac{512}{125}$ **k** $\frac{243}{32}$ **l** $\frac{32}{243}$

11 **a** x^4 **b** x^{-1} **c** $4y^2$
d $10x^2$ **e** $20x^{-1}$ **f** $\frac{1}{3}y$

12 **a** x **b** d^{-1} **c** $t^{\frac{3}{2}}$
d x^2 **e** $y^{\frac{1}{2}}$ **f** a^4

13 **a** $x^{\frac{1}{2}}$ **b** y^{-1} **c** $a^{\frac{5}{3}}$
d t^{-2} **e** d^2 **f** 1

14 $y^{\frac{9}{4}}$

Answers to Chapter 19

19.1 Direct proportion

Exercise 19A

1 **a** 15 **b** 2
2 **a** 75 **b** 6
3 **a** 150 **b** 6
4 **a** 22.5 **b** 12
5 **a** 175 kilometres **b** 8 hours
6 **a** 66.50 dollars **b** 175 kg
7 **a** 44 **b** 84 m^2
8 **a** 33 spaces
b 66 spaces since new car park has 366 spaces
9 17 minutes 30 seconds

Exercise 19B

1 **a** 100 **b** 10
2 **a** 27 **b** 5
3 **a** 56 **b** 1.69
4 **a** 192 **b** 2.25
5 **a** 25.6 **b** 5
6 **a** 80 **b** 8
7 **a** \$50 **b** 225
8 **a** 3.2°C **b** 10 atm
9 **a** 388.8 g **b** 3 mm
10 **a** 2 J **b** 40 m/s
11 **a** 78 dollars **b** 400 miles
12 4000 cm^3
13 \$250
14 **a** B **b** A **c** C
15 **a** B **b** A

19.2 Inverse proportion

Exercise 19C

1 $Tm = 12$ **a** 3 **b** 2.5
2 $Wx = 60$ **a** 20 **b** 6
3 $Q(5 - t) = 16$ **a** –3.2 **b** 4
4 $Mt^2 = 36$ **a** 4 **b** 5
5 $W\sqrt{T} = 24$ **a** 4.8 **b** 100
6 $x^3y = 32$ **a** 32 **b** 4
7 $gp = 1800$ **a** \$15 **b** 36
8 $td = 24$ **a** 3°C **b** 12 km
9 $ds^2 = 432$ **a** 1.92 km **b** 8 m/s
10 $p\sqrt{h} = 7.2$ **a** 2.4 atm **b** 100 m
11 $W\sqrt{F} = 0.5$ **a** 5 t/h **b** 0.58 t/h
12 B – This is inverse proportion, as x increases y decreases.
13

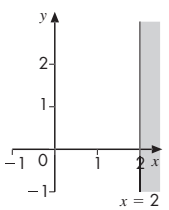
x	8	27	64
y	1	$\frac{2}{3}$	$\frac{1}{2}$

Answers to Chapter 20

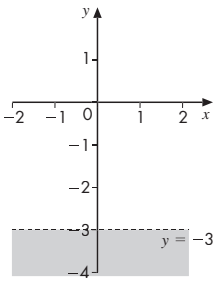
20.1 Graphical inequalities

Exercise 20A

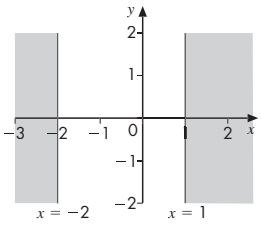
1 **a** & **b**



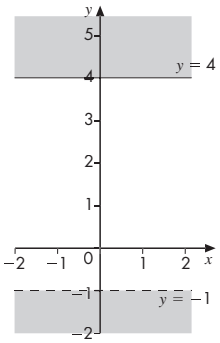
2 **a** & **b**



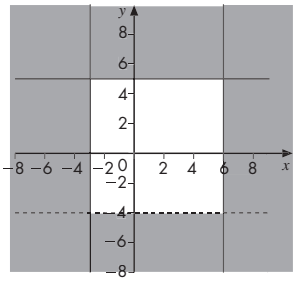
3 **a–c**



4 **a–c**

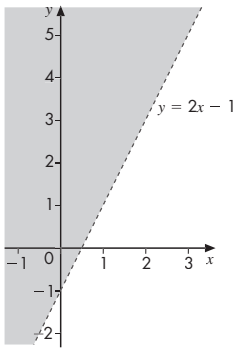


5 **a**

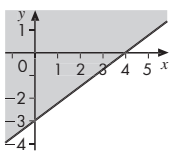


b **i** Yes **ii** Yes **iii** No

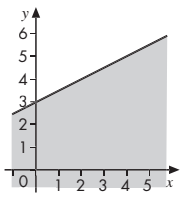
6 **a** & **b**



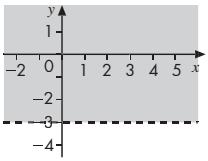
7 **a** & **b**



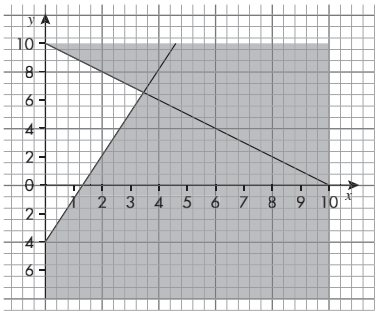
8 **a** & **b**



9



10 **a–d**

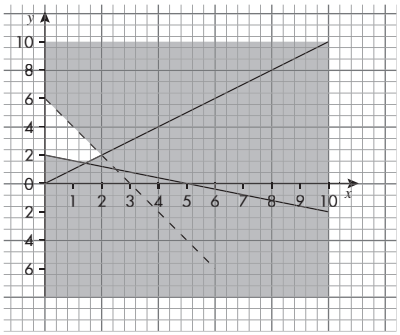


e **i** No **ii** Yes **iii** Yes

20.2 More than one inequality

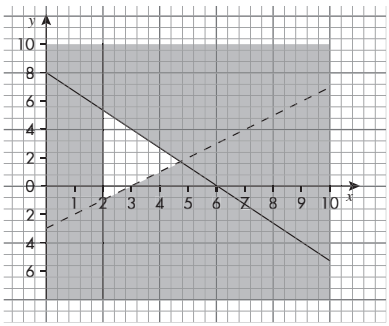
Exercise 20B

1 **a–f**



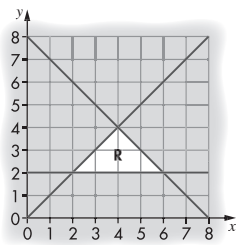
g **i** No **ii** No **iii** Yes

2 **a**

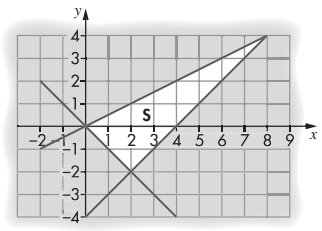


b **i** No **ii** Yes **iii** Yes **iv** No

3 **a** & **b**



4 **a** & **b**



c 4 **d** –2 **e** 12

5 Test a point such as the origin (0, 0), so $0 < 0 + 2$, which is true. So the side that includes the origin is the required side.

6 **a** $x + y \geqslant 3$, $y \leqslant \frac{1}{2}x + 3$ and $y \geqslant 5x - 15$

b 9 **c** 3 at (3,0)

7 $x \leqslant 4$ and $y \geqslant -1$ and $y < 3$

8 $x > 3$ and $y < 4$ and $y \geqslant x - 2$

9 $y \leqslant 4$ and $x + y > 0$ (or $y > -x$)

10 $x + y \geqslant 2$ and $x + y \leqslant 4$

11 $y < 4$ and $y < 2x$ and $y > 0.5x$

12 $y > 3$ and $y < x + 3$ and $x + y < 11$

13 $y < x + 3$and $y > x - 3$ and $x + y > 3$ and $x + y < 9$

Answers to Chapter 21

21.1 Function notation

Exercise 21A

1 **a** 12 **b** 26 **c** 7
d –2 **e** 3

2 **a** 0.5 **b** 5 **c** 50.5
d 2.5 **e** 0.625 or $\frac{5}{8}$

3 **a** 5 **b** –3 **c** 999801
d 1 **e** $\frac{1}{8}$

4 **a** 4 **b** 32 **c** 1
d $\frac{1}{2}$ **e** $\frac{1}{8}$

5 **a** 3 **b** 2 **c** 0
d –1 **e** 5

6 **a** 7.5 **b** –2.5 **c** –5

7 **a** 6 **b** 97 **c** 3.25

8 **a** 6 **b** at (6, 4)

9 **a** 3 **b** $-\frac{1}{2}$ **c** $x = 2$
d $x = 0$

21.2 Domain and range

Exercise 21B

1 $\{y: -23 \leqslant y \leqslant 17\}$

2 $\{y: 0.5 \leqslant y \leqslant 3\}$

3 $\{y: -2 \leqslant y \leqslant 10\}$

4 $\{y: 16 \leqslant y \leqslant 250\}$

5 **a** $\{x: -2 \leqslant x \leqslant 6\}$ **b** $\{y: 0 \leqslant y \leqslant 4\}$

6 **a** $\{x: x \geqslant 4\}$ **b** $\{y: y \geqslant 2\}$

7 **a** $\{x: -1 \leqslant x \leqslant 1\}$ **b** $\{y: 0 \leqslant y \leqslant 1\}$

8 1.25

9 $\{y: -16 \leqslant y \leqslant 0\}$

10 $\{y: 0 < y < 12\}$

11 $\{y: y \geqslant 0\}$

12 **a** $x = 3$ or -1 **b** $\{y: y \geqslant 2\}$

13 $\{y: y > 0\}$

14 $\{x: x \geqslant 8\}$

15 $\{x: x > 10\}$

21.3 Inverse functions

Exercise 21C

1 **a** $x - 7$ **b** $\frac{x}{8}$ **c** $5x$ **d** $x + 3$

2 **a** 8 **b** 4 **c** 5 **d** –2

3 **a** $3(x + 2)$ **b** $\frac{x}{4} + 5$ **c** $5x - 4$ **d** $\frac{5x + 6}{3}$
e $2(\frac{x}{3} - 4)$ **f** $\sqrt[3]{\frac{x}{4}}$

4 **a** 2 **b** $\frac{1}{2}$ **c** –2.5

5 **a** $10 - x$ **b** They are identical

6 **a** $\frac{8}{x}$ **b** $\frac{20}{x + 1}$ **c** $\frac{2}{x} - 1$

7 **a** $\frac{x + 4}{2}$ **b** student's graph **c** (4,4)

8 5

9 $\frac{x + 2}{3}$

21.4 Composite functions

Exercise 21D

1 **a** 6 and 3 **b** 7 and 3.5
c 10 and 5 **d** $\frac{x + 4}{2}$
e 1 and 5 **f** 1.5 and 5.5
g –5 and –1 **h** $\frac{x}{2} + 4$

2 **a** 6 and 216 **b** 10 and 1000 **c** $(2x)^3$ or $8x^3$
d 64 and 128 **e** $2x^3$

3 **a** 1, 9, 25 **b** 1, 3, 5 **c** $\sqrt{2x + 1}$

4 **a** 6 and 18 **b** 12 and 36 **c** $9x$

5 **a** $3(x - 6)$ **b** $3x - 6$

6 Both are $x - 3$

21.5 More about composite functions

Exercise 21E

1 **a** 3.5 **b** 1
c 8 **d** 5.5

2 **a** 20 **b** 9
c 8.75 **d** 3

3 **a** 7 **b** 8
c 256 **d** 21

4 **a** $6x$ **b** $6x - 5$

5 **a** $9x^2 + 24x + 16$ or $(3x + 4)^2$ **b** $6x - 5$
c $2x + 3$ **d** $4 - 2x$

6 **a** $x - 10$ **b** $x + 10$ **c** x

7 **a** x^4 **b** $\left(\frac{12}{x}\right)^2$ or $\frac{144}{x^2}$ **c** $\frac{12}{x^2}$ **d** x

8 **a** 80
b $2(2x - 1) - 1$ simplified
c $(2x - 1)^2 + 2(2x - 1)$ simplified

9 **a** $\frac{1}{3x - 3}$ **b** $\frac{x - 1}{x - 4}$

10 **a** $6x - 14$ **b** $\frac{x + 12}{4}$

11 **a** 0.5 (1 + 9) = 5 **b** 7 **c** 8
d 8.5, 8.75, 8.875, 8.9375, 8.96875, 8.984375
e Getting closer and closer to 9, halving the difference from 9 each time.

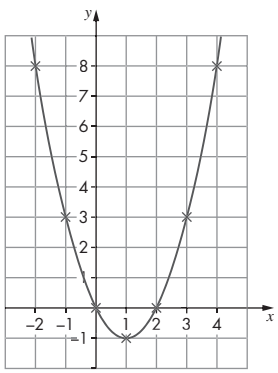
12 Student's own description of the convergence towards 9.

Answers to Chapter 22

22.1 The gradient of a curve

Exercise 22A

1 **a** The missing numbers are 0, –1, 0
b



c $2x - 2$ **d** 4
e 6 **f** Student's choice
g (1, –1) **h** Student's check

2 **a** $2x - 6$ **b** –6 **c** 4 **d** (4, 7)

3 **a** 4x **b** 8 **c** –4 **d** (3, 8)

4 **a** $4 - 2x$ **b** 4 and –4 **c** (1, 3) **d** (1.5, 3.75)

5 **a** $2x + 1$ **b** $2x - 7$ **c** $8x - 1$ **d** $0.6x - 1.5$
e –2 + 2x **f** $3 - 2x$ **g** 2 **h** 0

6 $2x + 2$

7 **a** $4x + 2$ **b** $2x + 7$ **c** $2x$

8 **a** (0, –5) **b** 2

22.2 More complex curves

Exercise 22B

1 **a** $6x^2$ **b** 6 and 24

2 **a** $3x^2 - 12x + 8$
b If $x = 0$ or 2 or 4, $y = 0$
c 8; –4; 8

3 **a** $1.5x^2 - 6x + 4$
b 4 at (0, 0) and (4, 0); –2 at (2,0)

4 **a** $8x^3$ **b** $6x^2 + 10x$
c $15x^2 - 2$ **d** $-1 - 2x^2$
e $9x^2 + 5$ **f** $-3x^2$
g $4x^3 - 1$ **h** $8x^3 + 18x^2$

5 16 at (2, 0); –16 at (–2, 0); 0 at (0, 0)

6 **a** $dy/dx = 4x^3 - 6x^2$ and if $x = 0$ then $dy/dx = 0$
b –10 **c** 8

7 $x^2 - 5 = 4$ has two solutions, $x = 3$ or -3.
Points are (3, –2) and (–3, 10)

8 $y = 1.5x - 2$

9 **a** 12 **b** 24

22.3 Turning points

Exercise 22C

1 **a** $2x - 4$ **b** $2x - 4 = 0 \Rightarrow x = 2$; (2, –1)
c Minimum

2 **a** (–3, –12) **b** Minimum

3 **a** $5 - 2x$ **b** (2.5, 7.25)
c Maximum

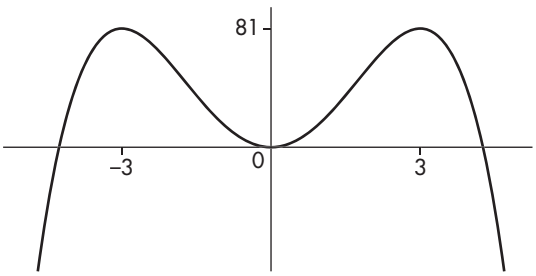
4 2 and –3

5 **a** $3x^2 - 6x$ **b** $x = 0$ or 2
c (0, 0) and (2, –4)

6 **a** If $x = -2$ or 5, $y = 0$ **b** $2x - 3$
c (1.5, –12.25); Minimum **d** $x = 1.5$

7 **a** (–3, 81), (0, 0) and (3, 81)

b



8 **a** $6x^2 - 6$ **b** (1, 0) minimum, (–1, 8) maximum

9 **a** The two sides add up to half the perimeter

b $15 - 2x$

c (7.5, 56.25)

d Maximum

e The largest possible area is 56.25 cm^2, when the rectangle is a square of side 7.5 cm

Answers to Chapter 23

23.1 Angle facts

Exercise 23A

1 **a** 48° **b** 307° **c** 108° **d** 52°
e 59° **f** 81° **g** 139° **h** 58°

2 **a** 82° **b** 105° **c** 75°

3 45° + 125° = 170° and for a straight line it should be 180°.

4 **a** $x = 100°$ **b** $x = 110°$ **c** $x = 30°$

5 **a** $x = 55°$ **b** $x = 45°$ **c** $x = 12.5°$

6 **a** $x = 34°$, $y = 98°$ **b** $x = 70°$, $y = 120°$ **c** $x = 20°$, $y = 80°$

7 6 × 60° = 360°; imagine six of the triangles meeting at a point.

8 $x = 35°$, $y = 75°$; $2x = 70°$ (opposite angles), so $x = 35°$ and $x + y = 110°$ (angles on a line), so $y = 75°$

9 $a = 88$ $b = 132$

23.2 Parallel lines

Exercise 23B

1 **a** 40° **b** $b = c = 70°$
c $d = 75$, $e = f = 105°$ **d** $g = 50°$, $h = i = 130°$
e $j = k = l = 70°$ **f** $n = m = 80°$

2 **a** $a = 50°$, $b = 130°$ **b** $c = d = 65°$, $e = f = 115°$
c $g = i = 65°$, $h = 115°$ **d** $j = k = 72°$, $l = 108°$
e $m = n = o = p = 105°$ **f** $q = r = s = 125°$

3 **a** $a = 95°$ **b** $b = 66°$, $c = 114°$

4 **a** $x = 30°$, $y = 120°$ **b** $x = 25°$, $y = 105°$
c $x = 30°$, $y = 100°$

5 **a** $x = 50°$, $y = 110°$ **b** $x = 25°$, $y = 55°$
c $x = 20$, $y = 140°$

6 290°; x is double the angle allied to 35°, so is 2 × 145°

7 $a = 66$

8 **a&b** Angle $PQD = 64°$ (alternate angles), so angle $DQY = 116°$ (angles on a line = 180°)

9 Use alternate angles to see b, a and c are all angles on a straight line, and so total 180°.

10 Third angle in triangle equals q (alternate angle), angle sum of triangle is 180°.

11 $A + D = 180°$ because they are allied angles. $C + D = 180°$ because they are allied angles. Hence $A = C$.

In the same way $B + C = 180° = D + C$ because they are pairs of allied angles. Hence $B = D$.

23.3 Angles in a triangle

Exercise 23C

1 **a** 70° **b** 50° **c** 80° **d** 60°
e 75° **f** 109° **g** 38° **h** 63°

2 **a** No, total is 190° **b** Yes, total is 180° **c** No, total is 170°
d Yes, total is 180° **e** Yes, total is 180° **f** No, total is 170°

3 **a** 60° **b** Equilateral triangle **c** Same length

4 **a** 70° each **b** Isosceles triangle **c** Same length

5 **a** 109° **b** 130° **c** 135°

6 Isosceles triangle; angle DFE ∠30° (opposite angles), angle DEF ∠75° (angles on a line), angle FDE ∠75° (angles in a triangle), so there are two equal angles in the triangle and hence it is an isosceles triangle

7 a is ∠80° (opposite angles), b is 65° (angles on a line), c is 35° (angles in a triangle)

8 Missing angle = y, $x + y = 180°$ and $a + b + y = 180°$ so $x = a + b$

9 $b = 240 - a$

23.4 Angles in a quadrilateral

Exercise 23D

1 **a** $a = 110°$, $b = 55°$ **b** $c = e = 105°$, $d = 75°$
c $f = 135°$, $g = 25°$ **d** $h = i = 94°$
e $j = l = 105°$, $k = 75°$ **f** $m = o = 49°$, $n = 131°$

2 **a** $x = 25°$, $y = 15°$ **b** $x = 7°$, $y = 31°$ **c** $x = 60°$, $y = 30°$

3 **a** $x = 50°$: 60°, 70°, 120°, 110° – possibly trapezium
b $x = 60°$: 50°, 130°, 50°, 130° – parallelogram or isosceles trapezium
c $x = 30°$: 20°, 60°, 140°, 140° – possibly kite
d $x = 20°$: 90°, 90°, 90°, 90° – square or rectangle

4 52°

5 Both 129°

6 $y = 360° - 4x$

7 36°, 72°, 108° and 144°

23.5 Regular polygons

Exercise 23E

1 **a** **i** 45° **ii** 8 **iii** 1080°
b **i** 20° **ii** 18 **iii** 2880°
c **i** 15° **ii** 24 **iii** 3960°
d **i** 36° **ii** 10 **iii** 1440°

2 **a** **i** 172° **ii** 45 **iii** 7740°
b **i** 174° **ii** 60 **iii** 10 440°
c **i** 156° **ii** 15 **iii** 2340°
d **i** 177° **ii** 120 **iii** 21 240°

3 **a** Exterior angle is 7°, which does not divide exactly into 360°
b Exterior angle is 19°, which does not divide exactly into 360°
c Exterior angle is 11°, which does divide exactly into 360°
d Exterior angle is 70°, which does not divide exactly into 360°

4 **a** 7° does not divide exactly into 360°
b 26° does not divide exactly into 360°
c 44° does not divide exactly into 360°
d 13° does not divide exactly into 360°

5 $x = 45°$, they are the same, true for all polygons

6 **a** 36° **b** 10

7 **a** The exterior angle is 180 – 170 = 10°; 360 ÷ 10 = 36 so a regular polygon with 36 sides is possible.
b The exterior angle is 180 – 169 = 11°; 360 ÷ 11 is not a whole number so a regular polygon is not possible.

23.6 Irregular polygons

Exercise 23F

1 **a** 1440° **b** 2340° **c** 17 640° **d** 7740°

2 **a** 9 **b** 15 **c** 102 **d** 50

3 **a** 130° **b** 95° **c** 130°

4 **a** 50° **b** 40° **c** 59°

5 Hexagon

6 **a** Octagon **b** 89°

7 **a** **i** 71° **ii** 109° **iii** Equal
b If S = sum of the two opposite interior angles, then $S + I = 180°$ (angles in a triangle), and we know $E + I = 180°$ (angles on a straight line), so $S + I = E + I$, therefore $S = E$

8 144°; 360 – (2 × 108)°

9 Three angles are 135° and two angles are 67.5°

23.7 Tangents and diameters

Exercise 23G

1 **a** 38° **b** 110° **c** 15° **d** 45°

2 **a** $x = 12°$, $y = 156°$ **b** $x = 100°$, $y = 50°$
c $x = 62°$, $y = 28°$ **d** $x = 30°$, $y = 60°$

3 **a** 90° **b** 62°

4 T is 90° because it is the angle in a semi-circle.
$R = 180° – (90° + 43°) = 47°$

5 **a** $x = 56°$ **b** $y = 25°$

6 $A = B = 45°$ and $C = 90°$

7 Angle $OCD = 58°$ (triangle OCD is isosceles), angle $OCB = 90°$ (tangent/radius theorem), so angle $DCB = 32°$, hence triangle BCD is isosceles (2 equal angles)

23.8 Angles in a circle

Exercise 23H

1 **a** 56° **b** 62° **c** 105° **d** 55°
e 45° **f** 30° **g** 60° **h** 145°

2 **a** 55° **b** 52° **c** 50° **d** 24°
e 39° **f** 80° **g** 34° **h** 30°

3 **a** 41° **b** 49° **c** 41°

4 **a** 72° **b** 37° **c** 72°

5 Angle $AZY = 35°$ (angles in a triangle), $a = 55°$ (angle in a semicircle = 90°)

6 **a** $x = y = 40°$ **b** $x = 131°$, $y = 111°$
c $x = 134°$, $y = 23°$ **d** $x = 32°$, $y = 19°$
e $x = 59°$, $y = 121°$ **f** $x = 155°$, $y = 12.5°$

7 68°

8 Angle $ABC = 180° – x$ (angles on a line), angle $AOC = 360° – 2x$ (angle at centre is twice angle at circumference), reflex angle $AOC = 360° – (360° – 2x) = 2x$ (angles at a point)

9 **a** x
b $2x$
c From part **b**, angle $AOD = 2x$
Similarly, angle $COD = 2y$
So angle $AOC = AOD + COD = 2x + 2y = 2(x + y)$
$= 2 \times$ angle ABC

23.9 Cyclic quadrilaterals

Exercise 23I

1 **a** $a = 50°$, $b = 95°$ **b** $c = 92°$, $x = 90°$
c $d = 110°$, $e = 110°$, $f = 70°$ **d** $g = 105°$, $h = 99°$
e $j = 89°$, $k = 89°$, $l = 91°$ **f** $m = 120°$, $n = 40°$
g $p = 44°$, $q = 68°$ **h** $x = 40°$, $y = 34°$

2 **a** $x = 26°$, $y = 128°$ **b** $x = 48°$, $y = 78°$
c $x = 133°$, $y = 47°$ **d** $x = 36°$, $y = 72°$
e $x = 55°$, $y = 125°$ **f** $x = 35°$
g $x = 48°$, $y = 45°$ **h** $x = 66°$, $y = 52°$

3 **a** $x = 49°$, $y = 49°$ **b** $x = 70°$, $y = 20°$
c $x = 80°$, $y = 100°$ **d** $x = 100°$, $y = 75°$

4 **a** $x = 50°$, $y = 62°$ **b** $x = 92°$, $y = 88°$
c $x = 93°$, $y = 42°$ **d** $x = 55°$, $y = 75°$

5 **a** $x = 95°$, $y = 138°$ **b** $x = 14°$, $y = 62°$
c $x = 32°$, $y = 48°$ **d** 52°

6 **a** 71° **b** 125.5° **c** 54.5°

7 **a** $x + 2x – 30° = 180°$ (opposite angles in a cyclic quadrilateral), so $3x – 30° = 180°$
b $x = 70°$, so $2x – 30° = 110°$ angle $DOB = 140°$ (angle at centre equals twice angle at circumference), $y = 80°$ (angles in a quadrilateral)

8 **a** x **b** $360° – 2x$
c Angle $ADC = \frac{1}{2}$reflex angle $AOC = 180° – x$, so angle ADC + angle $ABC = 180°$

9 Let angle $AED = x$, then angle $ABC = x$ (opposite angles are equal in a parallelogram), angle $ADC = 180° – x$ (opposite angles in a cyclic quadrilateral), so angle $ADE = x$ (angles on a line)

10 Let angle $ABC = x$ and angle $EFG = y$.
Then angle $ADC = 180 – x°$ (opposite angles in a cyclic quadrilateral) and angle $EDG = 180 – y°$.
But angle ADC = angle EDG (opposite angles).
$180 – x° = 180 – y°$ and therefore $x = y$.

23.10 Alternate segment theorem

Exercise 23J

1 **a** $a = 65°$, $b = 75°$, $c = 40°$
b $d = 79°$, $e = 58°$, $f = 43°$
c $g = 41°$, $h = 76°$, $i = 76°$
d $k = 80°$, $m = 52°$, $n = 80°$

2 **a** $a = 75°$, $b = 75°$, $c = 75°$, $d = 30°$

b $a = 47°$, $b = 86°$, $c = 86°$, $d = 47°$

c $a = 53°$, $b = 53°$

d $a = 55°$

3 **a** 36° **b** 70°

4 **a** $x = 25°$ **b** $x = 46°$, $y = 69°$, $z = 65°$

c $x = 38°$, $y = 70°$, $z = 20°$ **d** $x = 48°$, $y = 42°$

5 Angle $ACB = 64°$ (angle in alternate segment), angle $ACX = 116°$ (angles on a line), angle $CAX = 32°$ (angles in a triangle), so triangle ACX is isosceles (two equal angles)

6 Angle $AXY = 69°$ (tangents equal and so triangle AXY is isosceles), angle $XZY = 69°$ (alternate segment), angle $XYZ = 55°$ (angles in a triangle)

7 **a** $2x$ **b** $90° - x$ **c** angle $OPT = 90°$, so angle $APT = x$

Answers to Chapter 24

24.1 Measuring and drawing angles

Exercise 24A

1 **a** 40° **b** 125° **c** 340° **d** 225°

2 Student's drawings of angles

3 AC and BE; AD and CE; AE and CF.

4 Yes, the angle is 75°.

5 Any angle between 90° and 100°.

6 **a** 80° **b** 50° **c** 25°

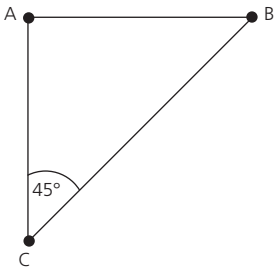
24.2 Bearings

Exercise 24B

1 **a** 110° **b** 250° **c** 091° **d** 270° **e** 130° **f** 180°

2 Student's sketches

3



4 **a** 090°, 180°, 270° **b** 000°, 270°, 180°

5

Leg	Actual distance	Bearing
1	50 km	060°
2	70 km	355°
3	65 km	260°
4	46 km	204°
5	60 km	130°

6 **a** 045° **b** 286°

7 **a** 250° **b** 325° **c** 144°

8 **a** 900 m **b** 280°

c angle $NHS = 150°$ and HS = 3 cm

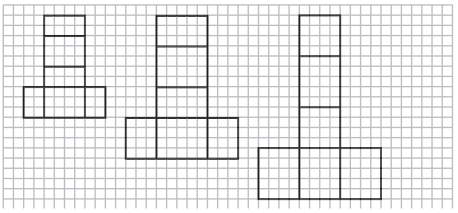
9 108°

10 255°

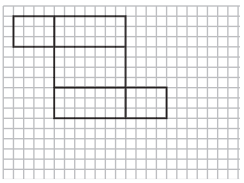
24.3 Nets

Exercise 24C

1 **a** **b** **c**



2 Yes.



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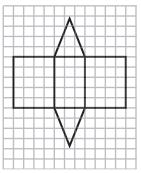
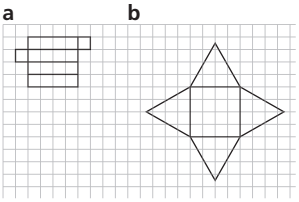
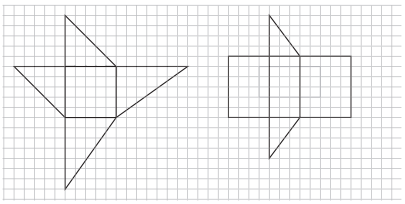


4 **a** **b**



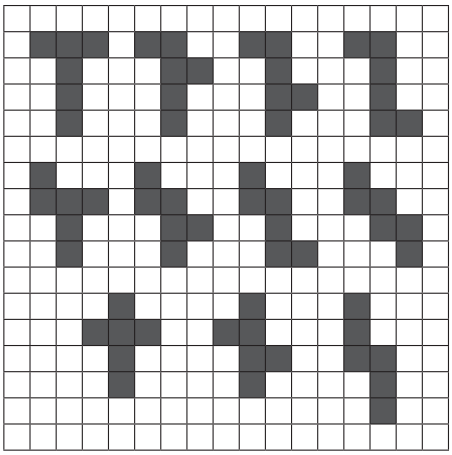
c **d**



5 **a** 488 cm^2 **b** 672 cm^3

6 **a** 2112 cm^2 **b** 6400 cm^3

7



8 a and b

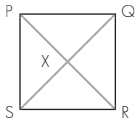
24.4 Congruent shapes

Exercise 24D

1 **a** yes **b** yes **c** no **d** yes **e** no **f** yes

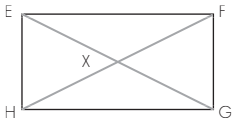
2 **a** triangle ii **b** triangle iii **c** sector i

3



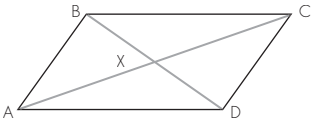
PQR to *QRS* to *RSP* to *SPQ*;
SXP to *PXQ* to *QXR* to *RXS*

4



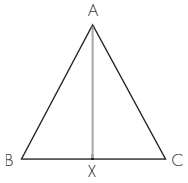
EGF to *FHE* to *GEH* to *HFG*;
EFX to *HGX*; *EXH* to *FXG*

5



ABC to *CDA*; *BDC* to *DBA*;
BXA to *DXC*; *BXC* to *DXA*

6



AXB to *AXC*

24.5 Similar shapes

Exercise 24E

1 **a** 2 **b** 3

2 **a** Yes, 4
b No, corresponding sides have different ratios.

3 **a** *PQR* is an enlargement of *ABC*
b 1 : 3 **c** Angle *R* **d** *BA*

4 **a** Sides in same ratio
b Angle *P* **c** *PR*

5 **a** Same angles **b** Angle *Q* **c** *AR*

6 **a** 8 cm **b** $x = 45$ cm, $y = 9$ cm
c $x = 19.5$ cm, $y = 10.8$ cm **d** 4.2 cm

7 **a** The angles are all 90 degrees. The sides of a square are all equal so the ratio between sides of two different squares will be the same, whatever two sides are chosen.
b No. They will only be similar if they have the same ratio of length to width.

8 5.2 m

24.6 Areas of similar shapes

Exercise 24F

1 **a** 2.5 **b** 125 cm^2

2 **a** All equilateral triangles are similar **b** 3.8 cm^2 (to 2 sf)

3 A is 62 cm^2 C is 558 cm^2

4 2880 cm^2

5 28 cm^2

6 40.32 cm^2

7 75 cm^2

8 **a** 144 cm^2 **b** 69.4 cm^2

9 **a** All angles are the same. **b** 247.7 cm^2

10 **a** 2 **b** 10 cm **c** 7.1 cm

11 354.9 cm^2

12 It will double the area.

13 28.3 cm^2

24.7 Areas and volumes of similar solids

Exercise 24G

1 **a** 4 : 25 **b** 8 : 125

2 **a** 16 : 49 **b** 64 : 343

3

Linear ratio	Linear scale factor	Area scale factor	Volume scale factor
1 : 2	2	4	8
1 : 3	3	9	27
4 : 1	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$
1 : 5	5	25	125
10 : 1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

4 135 cm^2

5 **a** 56 cm^2 **b** 126 cm^2

6 **a** 48 m^2 **b** 3 m^2

7 **a** 2400 cm^3 **b** 8100 cm^3

8 4 litres

9 1.38 m^3

10 \$6

11 4 cm

12 8 × 0.60 = \$4.80 which is greater than \$4.00 so the large tub is better value

13 **a** 3 : 4 **b** 9 : 16 **c** 27 : 64

14 720 ÷ 8 = 90 cm^3

Exercise 24H

1 6.2 cm, 10.1 cm

2 4.26 cm, 6.74 cm

3 9.56 cm

4 3.38 m
5 8.4 cm
6 26.5 cm
7 47.8 cm
8 **a** 4.33 cm, 7.81 cm **b** 143 g, 839 g
9 53.8 kg
10 1.73 kg
11 8.8 cm
12 7.9 cm and 12.6 cm
13 b

Answers to Chapter 25

25.1 Constructing shapes

Exercise 25A

1 **a** BC = 2.9 cm, angle B = 53°, angle C = 92°
b EF = 7.4 cm, ED = 6.8 cm, angle E = 50°
c angle G = 105°, angle H = 29°, angle I = 46°
d angle J = 48°, angle L = 32°, JK = 4.3 cm
e angle N = 55°, ON = OM = 7 cm
f angle P = 51°, angle R = 39°, QP = 5.7 cm
2 **a** Students can check one another's triangles.
b angle ABC = 44°, angle BCA = 79°, angle CAB = 57°
3 5.9 cm
4 **a** Student's drawing **b** 105°
5 **a** Student's drawing **b** 35°
6 **a** Student's drawing **b** 90°
7 **a** Student's drawing **b** 43°
8 **a** Student's drawing **b** 10.0 cm
9 Student's drawing
10 Student's drawing
11 4.3 cm
12 4.3 cm
13 **a** Right-angled triangle constructed with sides 3, 4, 5 and 4.5, 6, 7.5, and scale marked 1 cm : 1 m
b Right-angled triangle constructed with 12 equally spaced dots.
14 An equilateral triangle of side 4 cm.
15 Even with all three angles, you need to know at least one length.

25.2 Scale drawings

Exercise 25B

1 **a** pond: 40 m × 10 m, fruit: 50 m × 10 m, trees: 20 m × 20 m, lawn: 30 m × 20 m, vegetables: 50 m × 20 m
b pond: 400 m^2, fruit: 500 m^2, trees: 400 m^2, lawn: 600 m^2, vegetables: 1000 m^2
2 **a** 33 cm **b** 9 cm
3 **a** 30 cm × 30 cm **b** 40 cm × 10 cm **c** 20 cm × 15 cm
d 30 cm × 20 cm **e** 30 cm × 20 cm **f** 10 cm × 5 cm
4 **a** Student's scale drawing **b** 39 plants
5 **a** 8.4 km **b** 4.6 km **c** 6.2 km
d 6.4 km **e** 7.6 km **f** 2.4 km
6 **a** Student's drawing **b** 12.9 metres
7 **a** 900 km **b** 1100 km **c** 860 km
8 c – 7 cm represents 210 m, so 1 cm represents 30 m

Answers to Chapter 26

26.1 Pythagoras' theorem

Exercise 26A

1 10.3 cm
2 5.9 cm
3 8.5 cm
4 20.6 cm
5 18.6 cm
6 17.5 cm
7 5 cm
8 13 cm
9 10 cm
10 The smaller square in the first diagram and the two smaller squares in the second have the same area.

Exercise 26B

1 **a** 15 cm **b** 14.7 cm **c** 6.3 cm **d** 18.3 cm
2 **a** 20.8 m **b** 15.5 cm **c** 15.5 m **d** 12.4 cm
3 **a** 5 m **b** 6 m **c** 3 m **d** 50 cm
4 There are infinite possibilities, e.g. any multiple of 3, 4, 5 such as 6, 8, 10; 9, 12, 15; 12, 16, 20; multiples of 5, 12, 13 and of 8, 15, 17.
5 42.6 cm

26.2 Trigonometric ratios

Exercise 26C

1 **a** 0.682 **b** 0.829 **c** 0.922 **d** 1
2 **a** 0.731 **b** 0.559 **c** 0.388 **d** 0
3 **a** **i** 0.574 **ii** 0.574
b **i** 0.208 **ii** 0.208
c **i** 0.391 **ii** 0.391
d They are the same.
e **i** sin 15° is the same as cos 75°
ii cos 82° is the same as sin 8°
iii sin x is the same as cos (90° – x)
4 **a** 0.933 **b** 1.48 **c** 2.38 **d** Infinite
e 1 **f** 0.364 **g** 0.404 **h** 0
5 Tan has values > 1
6 **a** 3.56 **b** 8.96 **c** 28.4 **d** 8.91
7 **a** 5.61 **b** 7.08 **c** 1.46 **d** 7.77
8 **a** $\frac{4}{5}, \frac{3}{5}, \frac{4}{3}$ **b** $\frac{5}{13}, \frac{12}{13}, \frac{5}{12}$ **c** $\frac{7}{25}, \frac{24}{25}, \frac{7}{24}$

26.3 Calculating angles

Exercise 26D

1 **a** 30° **b** 51.7° **c** 39.8°
d 61.3° **e** 87.4° **f** 45.0°

2 **a** 60° **b** 50.2° **c** 2.6°
d 45° **e** 78.5° **f** 45.6°

3 **a** 31.0° **b** 20.8° **c** 41.8°
d 46.4° **e** 69.5° **f** 77.1°

4 Error message, largest value 1, smallest value –1

5 **a** **i** 17.5° **ii** 72.5° **iii** 90°
b Yes

26.4 Using sine, cosine and tangent functions

Exercise 26E

1 **a** 17.5° **b** 22.0° **c** 32.2°

2 **a** 5.29 cm **b** 5.75 cm **c** 13.2 cm

3 **a** 4.57 cm **b** 6.86 cm **c** 100 cm

4 **a** 5.12 cm **b** 9.77 cm **c** 11.7 cm **d** 15.5 cm

Exercise 26F

1 **a** 51.3° **b** 75.5° **c** 51.3°

2 **a** 5.35 cm **b** 14.8 cm **c** 12.0 cm **d** 8.62 cm

3 **a** 5.59 cm **b** 46.6° **c** 9.91 cm **d** 40.1°

Exercise 26G

1 **a** 33.7° **b** 36.9° **c** 52.1°

2 **a** 9.02 cm **b** 7.51 cm **c** 7.14 cm **d** 8.90 cm

3 **a** 13.7 cm **b** 48.4° **c** 7.03 cm **d** 41.2°

26.5 Which ratio to use

Exercise 26H

1 **a** 12.6 **b** 59.6 **c** 74.7 **d** 16.0
e 67.9 **f** 20.1

2 **a** 44.4° **b** 39.8° **c** 44.4° **d** 49.5°
e 58.7° **f** 38.7°

3 **a** 67.4° **b** 11.3 **c** 134 **d** 28.1°
e 39.7 **f** 263 **g** 50.2° **h** 51.3°
i 138 **j** 22.8

4 **a** Sides of right-hand triangle are sine θ and cosine θ
b Pythagoras' theorem
c Students should check the formulas.

26.6 Exact values of trigonometric ratios

Exercise 26I

1 **a** $\frac{1}{\sqrt{2}}$ **b** $\frac{1}{\sqrt{2}}$ **c** 1

2 **a** $\frac{\sqrt{3}}{2}$ **b** $\frac{1}{2}$ **c** $\sqrt{3}$

3 **a** $\frac{1}{2}$ **b** $\frac{\sqrt{3}}{2}$ **c** $\frac{1}{\sqrt{3}}$

4

	30°	45°	60°
Sine	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
Cosine	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
Tangent	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

5 **a** 10 cm **b** $10\sqrt{3}$ cm

6 **a** 6 cm **b** $6\sqrt{3}$ cm

7 **a** $BC = 8\sin 45^\circ = 8 \times \frac{1}{\sqrt{2}} = \frac{8}{\sqrt{2}}$ cm

b $BC = \frac{8}{\sqrt{2}} = \frac{8 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{8\sqrt{2}}{2} = 4\sqrt{2}$ cm

c Area $= \frac{1}{2} \times AC \times BC = \frac{1}{2} \times 4\sqrt{2} \times 4\sqrt{2} = \frac{1}{2} \times 16 \times 2 = 16\text{ cm}^2$

8 **a** $YZ = 6\tan 30^\circ = 6 \times \frac{1}{\sqrt{3}} = \frac{6}{\sqrt{3}}$ cm

b $YZ = \frac{6}{\sqrt{3}} = \frac{6 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$ cm

c Area $= \frac{1}{2} \times XZ \times YZ = \frac{1}{2} \times 6 \times 2\sqrt{3} = 6\sqrt{3}\text{ cm}^2$

9 **a** $BH = 10\sin 45^\circ = 10 \times \frac{1}{\sqrt{2}} = \frac{10}{\sqrt{2}} = \frac{10 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{10\sqrt{2}}{2}$
$= 5\sqrt{2}$ cm

b $\tan 60^\circ = \frac{HC}{BH}$ so $HC = BH \times \tan 60^\circ = 5\sqrt{2} \times \sqrt{3} = 5\sqrt{6}$ cm

10 **a** $\sin 30^\circ + \cos 30^\circ = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$

b $\sin 60^\circ + \cos 60^\circ = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{1+\sqrt{3}}{2}$ (the same as part **a**)

c $\tan 30^\circ + \tan 60^\circ = \frac{1}{\sqrt{3}} + \sqrt{3} = \frac{\sqrt{3}}{\sqrt{3} \times \sqrt{3}} + \sqrt{3} = \frac{\sqrt{3}}{3} + \sqrt{3}$ =

26.7 Application of trigonometric ratios

Exercise 26J

1 14.0 cm

2 **a** 24.5 cm **b** 20.6 cm **c** 19.4 cm

3 1.46 km

4 3.33

5 10.1 km

6 22°

7 429 m

8 **a** 156 m
b No. the new angle of depression is $\tan^{-1}\left(\frac{200}{312}\right) = 33°$ and half of 52° is 26°

9 **a** 222 m **b** 42°

10 **a** 21.5 m **b** 17.8 m

11 13.4 m
12 19°
13 The angle is 16° so Cara is not quite correct.

26.8 Problems in three dimensions

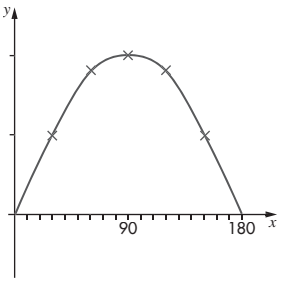
Exercise 26K

1 25.1°
2 **a** 25cm **b** 58.6° **c** 20.5 cm
3 **a** 3.46 m **b** 75.5° **c** 73.2°
4 **a** 24.0° **b** 48.0° **c** 13.5 cm **d** 16.6°
5 It is 44.6°; use triangle XDM where M is the midpoint of BD; triangle DXB is isosceles, as X is over the point where the diagonals of the base cross; the length of DB is $\sqrt{656}$. and the cosine of the required angle is $0.5\sqrt{656} \div 18$

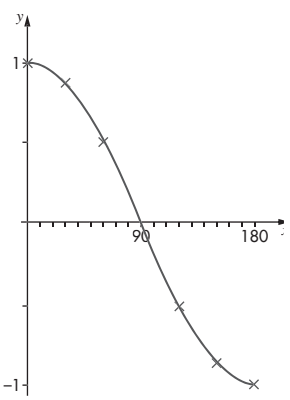
26.9 Sine and cosine of obtuse angles

Exercise 26L

1 **a** The bottom row of the table is 0.174, 0.5, 0.766, 0.996, 1, 0.996, 0.766, 0.5, 0.174.
b



c It has reflection symmetry. The line of symmetry is $x = 90$.
d You could choose 10° and 170°, 30° and 150°, 50° and 130° or 85° and 95°
2 30° and 150°.
3 46° and 134°.
4 122.9°
5 **a** The bottom row of the table is 0.966, 0.819, 0.5, 0.174, 0, –0.174, –0.5, –0.819, –0.966.
b



c It has rotational symmetry of order 2 about the point (90, 0)
6 **a** 31.8° **b** 148.2° **c** 120°
d 90° **e** 82.8° **f** 97.2°
7 **a** 53° **b** 104° **c** 49°, 131° **d** 90°
e 90° **f** 72°, 108° **g** no solution **h** 45°

26.10 The sine rule and the cosine rule

Exercise 26M

1 **a** 3.64 m **b** 8.05 cm **c** 19.4 cm
2 **a** 46.6° **b** 68° **c** 36.2°
3 **a** **i** 30° **ii** 40°
b 19.4 m
4 36.5 m
5 22.2 m
6 3.47 m
7 64.6 km
8 134°

Exercise 26N

1 **a** 7.71 m **b** 29.1 cm **c** 27.4 cm
2 **a** 76.2° **b** 125.1° **c** 90°
d Right-angled triangle
3 5.16 cm
4 65.5 cm
5 **a** 10.7 cm **b** 41.7° **c** 38.3° **d** 6.69 cm
6 58.4 km at 092.5°
7 21.8°
8 42.5 km
9 111°; the largest angle is opposite the longest side

Exercise 26O

1 **a** 8.60 m **b** 90° **c** 27.2 cm
d 26.9° **e** 27.5° **f** 62.4 cm
g 90.0° **h** 866 cm **i** 86.6 cm
2 7 cm
3 11.1 km
4 **a** BAC = 90°; this is Pythagoras' theorem
b BAC is acute
c BAC is obtuse
5 142 m

26.11 Using sine to find the area of a triangle

Exercise 26P

1 **a** 24.0 cm^2 **b** 26.7 cm^2 **c** 243 cm^2
d 21 097 cm^2 **e** 1224 cm^2
2 4.26 cm
3 **a** 42.3° **b** 49.6°
4 2033 cm^2
5 21.0 cm^2
6 726 cm^2
7 149 km^2
8 **a** 66.4 m **b** 118.9° **c** 1470 m^2
9 43.3 cm^2

26.12 Sine, cosine and tangent of any angle

Exercise 26Q

1 **a** 100° **b** 34° **c** 325° **d** 234°

2 **a** 350° **b** 235° **c** 152° **d** 49°
3 **a** 27° and 153° **b** 56° and 124°
c 333° and 207° **d** 304° and 236°
4 **a** 37° and 323° **b** 103° and 257°
c 157° and 203° **d** 85° and 275°
5 **a** 30° and 150° **b** 60° and 120°
c 225° and 315° **d** 270°
6 **a** 120° and 240° **b** 30° and 330°
c 45° and 315° **d** 90° and 270°
7 **a** 87.1° and 272.9° **b** 54.3° and 124.7°
c 130.5° and 229.5° **d** 323.1° and 216.9°
8 **a** 41.8° and 138.2° **b** 36.9° and 323.1°
c 314.4° and 225.6°
9 540°
10 30°, 150°, 210° and 330°
11 45°, 135°, 225° and 315°

Exercise 26R

1 **a** 215° **b** 265° **c** 298°
d 20° **e** 63° **f** 157°
2 **a** 45° and 225° **b** 135° and 315°
c 60° and 240° **d** 120° and 300°
3 **a** 11.3° and 191.3° **b** 78.7° and 258.7°
c 160.7° and 340.7° **d** 103.5° and 283.5°
4 –1.5
5 **a** 20.6° and 200.6° **b** 69.4° and 249.4°
c 144.2° and 324.2° **b** 102.7° and 282.7°
6 **a** 45°, 135°, 225° and 315° **b** 60°, 120°, 240° and 300°
c 30°, 150°, 210° and 330°
7 71.6°, 251.6°, 104.0° and 284.0°
8 **a** They are the same. **b** and **c** They have the same magnitude but different signs. They add up to 0.

Answers to Chapter 27

27.1 Perimeter and area of a rectangle

Exercise 27A

1 **a** 35 cm^2, 24 cm **b** 33 cm^2, 28 cm
c 45 cm^2, 36 cm **d** 70 cm^2, 34 cm
e 56 cm^2, 30 cm **f** 10 cm^2, 14 cm
2 **a** 53.3 cm^2, 29.4 cm **b** 84.96 cm^2, 38 cm
3 39
4 **a** 4 **b** 1 h 52 min
5 40 cm
6 Area B, 44 cm^2; perimeter B, 30 cm
7 Never (the area becomes four times greater).
8 **a** 28 cm, 30 cm^2 **b** 28 cm, 40 cm^2
c 40 cm, 51 cm^2 **d** 30 cm, 35 cm^2
e 32 cm, 43 cm^2 **f** 34 cm, 51 cm^2
g cannot tell what the perimeter is; 48 cm^2
h 34 cm, 33 cm^2
9 72 cm^2
10 48 cm

27.2 Area of a triangle

Exercise 27B

1 **a** 21 cm^2 **b** 12 cm^2 **c** 14 cm^2
d 55 cm^2 **e** 90 cm^2 **f** 140 cm^2
2 **a** 28 cm^2 **b** 8 cm **c** 4 cm
d 3 cm **e** 7 cm **f** 44 cm^2
3 73.9 cm^2
4 **a** 40 cm^2 **b** 65 m^2 **c** 80 cm^2
5 **a** 65 cm^2 **b** 50 m^2
6 For example: height 10 cm, base 10 cm; height 5 cm, base 20 cm; height 25 cm, base 4 cm; height 50 cm, base 2 cm
7 Triangle c; a and b each have an area of 15 cm^2 but c has an area of 16 cm^2

27.3 Area of a parallelogram

Exercise 27C

1 **a** 96 cm^2 **b** 70 cm^2 **c** 20 m^2
d 125 cm^2 **e** 10 cm^2 **f** 112 m^2
2 No, it is 24 cm^2, she used the slanting side instead of the perpendicular height.
3 16 cm
4 **a** 500 cm^2 **b** 15

27.4 Area of a trapezium

Exercise 27D

1 **a** 30 cm^2 **b** 77 cm^2 **c** 24 cm^2 **d** 42 cm^2
e 40 cm^2 **f** 6 cm **g** 3 cm
2 **a** 27.5 cm, 36.25 cm^2
b 33.4 cm, 61.2 cm^2
c 38.6 m, 88.2 m^2
3 Any pair of lengths that add up to 10 cm. For example: 1 cm, 9 cm; 2 cm, 8 cm; 3 cm, 7 cm; 4 cm, 6 cm; 4.5 cm, 5.5 cm
4 Shape c. Its area is 25.5 cm^2
5 Shape a. Its area is 28 cm^2
6 a
7 2 cm
8 1.4 m^2

27.5 Circumference and area of a circle

Exercise 27E

1 **a** 10π cm and 25π cm^2 **b** 6π cm and 9π cm^2
c 3π cm and 2.25π cm^2 **d** 8π cm and 16π cm^2
2 **a** 25.1 cm and 50.3 cm^2
b 15.7 cm and 19.6 cm^2
c 28.9 cm and 66.5 cm^2
d 14.8 cm and 17.3 cm^2

3 **a** **i** 56.5 cm **ii** 81π, 254.5 cm^2
b **i** 69.1 cm **ii** 121π, 380.1 cm^2
c **i** 40.8 cm **ii** 42.3π, 132.7 cm^2
d **i** 88.0 cm **ii** 196π, 615.8 cm^2

4 **a** 19.1 cm **b** 9.5 cm
c 286.5 cm^2 (or 283.5 cm^2)

5 962.9 cm^2 (or 962.1 cm^2)

6 **a** 20 cm **b** 400π cm^2

7 **a** 16π m^2 **b** 14π cm^2 **c** 9π cm^2

8 45π cm^2

9 $a^2 = \pi r^2$, so $r^2 = \frac{a^2}{\pi}$ therefore $r = \frac{a}{\sqrt{\pi}}$

10 21.5 cm^2

27.6 Surface area and volume of a cuboid

Exercise 27F

1 **a** **i** 198 cm^3 **ii** 234 cm^2
b **i** 90 cm^3 **ii** 146 cm^2
c **i** 1440 cm^3 **ii** 792 cm^2
d **i** 525 cm^3 **ii** 470 cm^2

2 24 litres

3 **a** 160 cm^3 **b** 416 cm^3 **c** 150 cm^3

4 **a** **i** 64 cm^3 **ii** 96 cm^2
b **i** 343 cm^3 **ii** 294 cm^2
c **i** 1000 mm^3 **ii** 600 mm^2
d **i** 125 m^3 **ii** 150 m^2
e **i** 1728 m^3 **ii** 864 m^2

5 86

6 **a** 180 cm^3 **b** 5 cm **c** 6 cm
d 10 cm **e** 81 cm^3

7 1.6 m

8 48 m^2

9 **a** 3 cm **b** 5 m **c** 2 mm **d** 1.2 m

10 **a** 148 cm^3 **b** 468 cm^3

11 If this was a cube, the side length would be 5 cm, so total surface area would be 5 × 5 × 6 = 150 cm^2; no this particular cuboid is not a cube.

12 **a** 6 cm **b** 216

27.7 Volume and surface area of a prism

Exercise 27G

1 **a** **i** 21 cm^2 **ii** 63 cm^3
b **i** 48 cm^2 **ii** 432 cm^3
c **i** 36 m^2 **ii** 324 m^3

2 **a** 432 m^3 **b** 225 m^3 **c** 1332 m^3

3 **a** A cross-section parallel to the side of the pool always has the same shape.
b About $3\frac{1}{2}$ hours

4 7.65 m^3

5 **a** 21 cm^3, 210 cm^3
b 54 cm^2, 270 cm^2

6 146 cm^3

7 78 m^3 (78.3 m^3)

8 327 litres

9 10.2 tonnes

10 **a** 672 cm^3
b $x = 12$, $y = 12$, $z = 20$
c 528 cm^2

27.8 Volume and surface area of a cylinder

Exercise 27H

1 **a** **i** 72π cm^3 **ii** 66π cm^2
b **i** 4.75π cm^3 **ii** 19.5π cm^2
c **i** 110π cm^3 **ii** 87.5π cm^2
d **i** 338π cm^3 **ii** 203π cm^2

2 **a** **i** 72π cm^3 **ii** 48π cm^2
b **i** 112π cm^3 **ii** 56π cm^2
c **i** 180π cm^3 **ii** 60π cm^2
d **i** 600π m^3 **ii** 120π m^2

3 665 cm^3

4 Label should be less than 10.5 cm wide so that it fits the can and does not overlap the rim and more than 23.3 cm long to allow an overlap.

5 332 litres

6 There is no right answer. Students could start with the dimensions of a real can. Often drinks cans are not exactly cylindrical. One possible answer is height of 6.6 cm and diameter of 8 cm.

7 About 127 cm

8 A diameter of 10 cm and a length of 5 cm give a volume close to 400 cm^3 (0.4 litres).

27.9 Sectors and arcs: 1

Exercise 27I

1 **a** 20π cm **b** **i** 10π cm **ii** 5π cm **iii** 2.5π cm

2 **a** 100π cm² **b** **i** 50π cm² **ii** 25π cm² **iii** 12.5π cm²

3 **a** $\frac{1}{5}$ **b** 10.6 cm **c** 44.3 cm²

4 **a** 96.5 cm² **b** 20.1 cm **c** 39.3 cm

5 **a** 245.4 cm² to 1 d.p. **b** 64.3 cm to 1 d.p.

6 **a** The diameter is 80 and the fraction of a circle is $\frac{1}{10}$.
The arc length is π × 80 ÷ 10 = 8π cm.
b 160π cm²

7 **a** 20π **b** 75π cm²

27.10 Sectors and arcs: 2

Exercise 27J

1 **a** **i** 5.59 cm **ii** 22.3 cm^2
b **i** 8.29 cm **ii** 20.7 cm^2
c **i** 16.3 cm **ii** 98.0 cm^2
d **i** 15.9 cm **ii** 55.6 cm^2

2 **a** 9π cm **b** 54π cm^2

3 **a** 73.8 cm **b** 20.3 cm

4 **a** 107 cm^2 **b** 173 cm^2

5 43.6 cm

6 (36π – 72) cm^2

7 (32π – 64)

8 **a** 13.9 cm **b** 7.07 cm^2

27.11 Volume of a pyramid

Exercise 27K

1 **a** 56 cm^3 **b** 168 cm^3 **c** 1040 cm^3
d 84 cm^3 **e** 160 cm^3

2 270 cm^3

3 **a** Put the apexes of the pyramids together. The 6 square bases will then form a cube.
b If the side of the base is a then the height will be $\frac{1}{2}a$.
Total volume of the 6 pyramids is a^3.
Volume of one pyramid is
$\frac{1}{6}a^3 = \frac{1}{3} \times \frac{1}{2} \times a \times a^2 = \frac{1}{3} \times$ height × base area

4 6.9 m (height of cuboid)

5 **a** 73.3 m^3 **b** 45 m^3 **c** 3250 cm^3

6 1.5 g

7 5.95 cm

8 14.4 cm

9 260 cm^3

10 **a** 96 m^2 **b** 48 m^3

11 1460 cm^2

27.12 Volume and surface area of a cone

Exercise 27L

1 **a** **i** 3560 cm^3 **ii** 1430 cm^2
b **i** 314 cm^3 **ii** 283 cm^2
c **i** 1020 cm^3 **ii** 679 cm^2

2 24π cm^2

3 **a** 816π cm^3 **b** 720π mm^3

4 **a** 4 cm **b** 6 cm
c Various answers, e.g. 60° gives 2 cm, 240° gives 8 cm

5 24π cm^2

6 If radius of base is r, slant height is $2r$.
Area of curved surface = $\pi r \times 2r = 2\pi r^2$, area of base = πr^2

7 140 g

8 2.81 cm

27.13 Volume and surface of a sphere

Exercise 27M

1 **a** 36π cm^3 and 36 π cm^2
b 288π cm^3 and 144 π cm^2
c 1330π cm^3 and 400 π cm^2

2 65 400 cm^3, 7850 cm^2

3 **a** 1960 cm^2
b 8180 cm^3

4 125 cm

5 6231

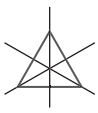
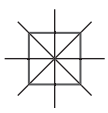
6 7.8 cm

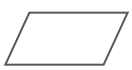
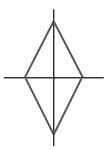
7 48%

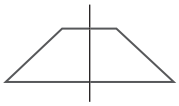
Answers to Chapter 28

28.1 Lines of symmetry

Exercise 28A

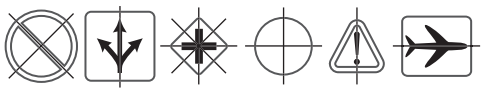
1 **a**  **b**  **c** 

d  **e**  **f** 

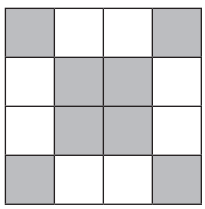
g 

2 **a** **i** 5 **ii** 6 **iii** 8
b 10

3 2, 1, 1, 2, 0

4 

5 **a** 1 **b** 5 **c** 1 **d** 6

6 

28.2 Rotational symmetry

Exercise 28B

1 **a** 4 **b** 2 **c** 2 **d** 3 **e** 6

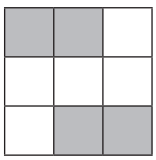
2 **a** 4 **b** 5 **c** 6 **d** 4 **e** 6

3 **a** 2 **b** 2 **c** 2 **d** 2 **e** 2

4 **a** 6

b 9 (the small red circle surrounded by nine 'petals') and 12 (the centre pattern)

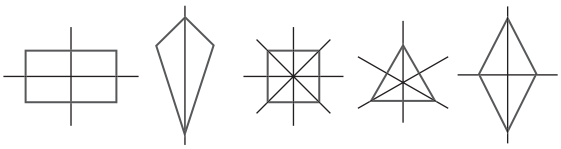
5 For example:



28.3 Symmetry of special two-dimensional shapes

Exercise 28C

1



2 **a** kite

b rectangle 2, square 4, equilateral triangle 3, rhombus 2

3 **a** isosceles **b** no

4 **a** parallelogram **b** square

5 **a** rectangle and rhombus **b** no

6 **a** B and D **b** AB and AD; CB and CD **c** kite

7 **a** diameter **b** infinite **c** infinite

8 **a** A and C; B and D

b AD and BC; AB and DC

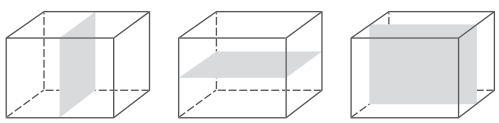
c Parallelogram

9 It will have two pairs of equal angles

28.4 Symmetry of three-dimensional shapes

Exercise 28D

1



2 **a** Diagrams to show axes going through the centres of all three pairs of opposite faces

b 2

3 Two are similar to the one shown, dividing the end triangles in two. The other goes through the centre of each of the long edges, parallel to the end triangles.

4 **a** 3 about AB; 2 about CD

b they are similar to CD, each passing through the centre of a rectangular face.

5 **a** 4 **b** no

6 There are four. All pass through the vertex. Two pass through opposite corners of the square. Two pass through the mid points of opposite sides of the square

7 **a** Any plane dividing each circle in half or the circular plane exactly half way up the cylinder

b any line at right angles to the one shown, passing through the centre of the cylinder

8 **a** one **b** infinite

9 **a** six through the centre of each hexagon; one parallel to the hexagons passing through the centre of the prism

b 4

28.5 Symmetry in circles

Exercise 28E

1 **a–d** Student's own drawing

e because the perpendicular bisector of any chord passes through the centre of the circle

f Here is one method: draw two chords; construct the perpendicular bisectors; they meet at the centre

2 **a** Isosceles because OA and OB are radii

b OA = OC; OB = OD; AB = CD so corresponding sides are equal

c 50°

3 **a** EM = FM (given); OE = OF (radii); OM is common to both. Corresponding sides are equal

b EMO and FMO are equal and add up to 180° (because EMF is a straight line) so they must both be 90°

c 18°

4 **a** Angle between a radius and a tangent

b XP = YP (tangents from a point are equal); OX = OY (radii); OP is common. So corresponding sides are equal

c 146°

5 5 cm (use Pythagoras' theorem)

6 40°

Answers to Chapter 29

29.1 Translations

Exercise 29A

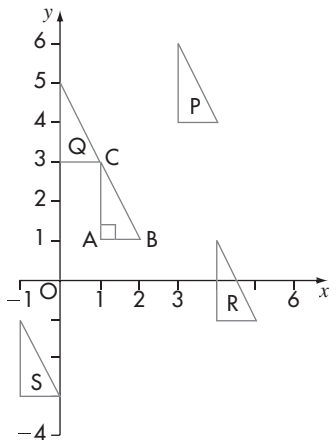
1 **a** **i** $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ **ii** $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$ **iii** $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$

b **i** $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$ **ii** $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ **iii** $\begin{pmatrix} 3 \\ 3 \end{pmatrix}$

c **i** $\begin{pmatrix} -4 \\ -2 \end{pmatrix}$ **ii** $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ **iii** $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$

d i $\begin{pmatrix}-2\\-7\end{pmatrix}$ **ii** $\begin{pmatrix}5\\0\end{pmatrix}$ **iii** $\begin{pmatrix}1\\-5\end{pmatrix}$

2



3 **a** $\begin{pmatrix}-3\\-1\end{pmatrix}$ **b** $\begin{pmatrix}4\\-4\end{pmatrix}$ **c** $\begin{pmatrix}-5\\-2\end{pmatrix}$

d $\begin{pmatrix}4\\7\end{pmatrix}$ **e** $\begin{pmatrix}-1\\5\end{pmatrix}$ **f** $\begin{pmatrix}1\\6\end{pmatrix}$

g $\begin{pmatrix}-4\\4\end{pmatrix}$ **h** $\begin{pmatrix}-4\\-7\end{pmatrix}$

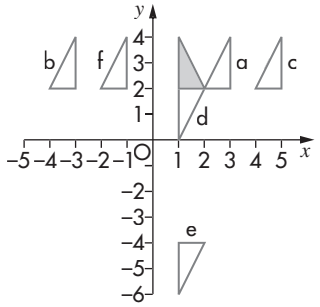
4 $\begin{pmatrix}-x\\-y\end{pmatrix}$

5 $\begin{pmatrix}-1\\4\end{pmatrix}$

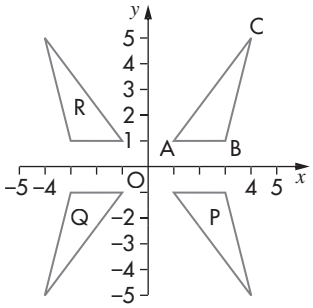
29.2 Reflections: 1

Exercise 29B

1

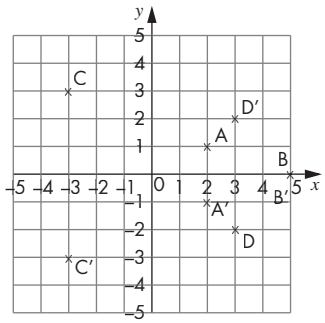


2 **a–e**



f Reflection in the y-axis

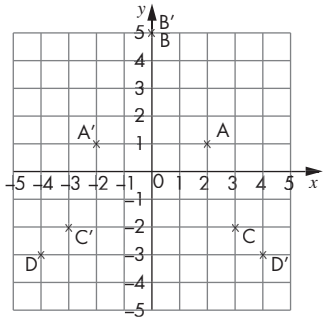
3 **a–b**



c y-value changes sign

d $(a, -b)$

4 **a–b**



c x-value changes sign

d $(-a, b)$

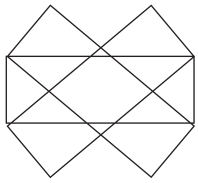
5 Possible answer: Take the centre square as ABCD then reflect this square each time in the line, AB, then BC, then CD and finally AD.

6 $x = -1$

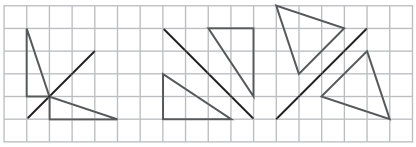
29.3 Reflections: 2

Exercise 29C

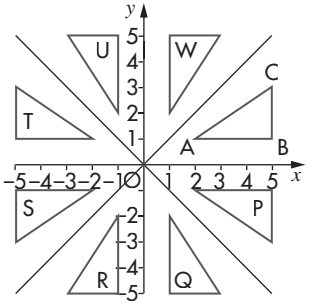
1 Possible answer:



2

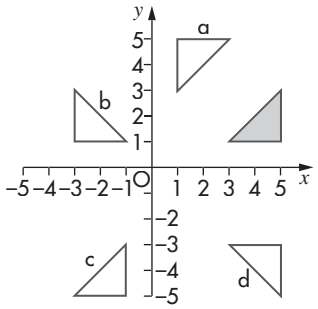


3 **a–i**

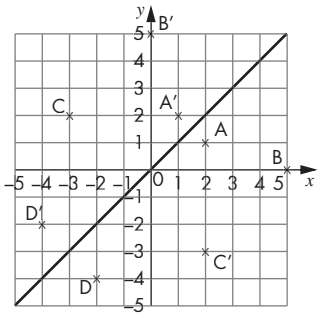


j A reflection in $y = x$

4



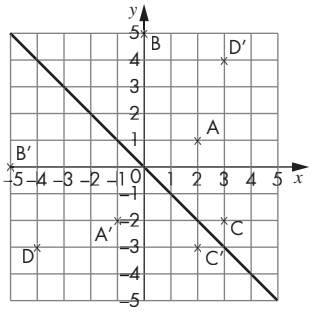
5 **a–c**



d Coordinates are reversed: x becomes y and y becomes x

e (b, a)

6 **a–c**



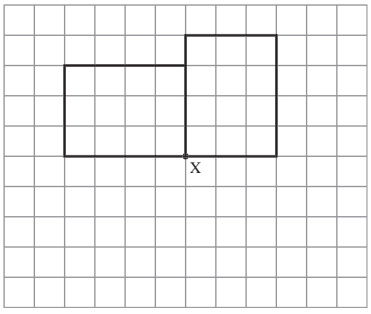
d Coordinates are reversed and the sign changes: x becomes $-y$ and y becomes $-x$

e $(-b, -a)$

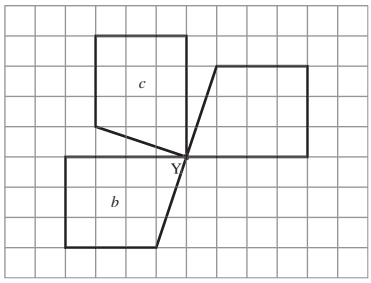
29.4 Rotations: 1

Exercise 29D

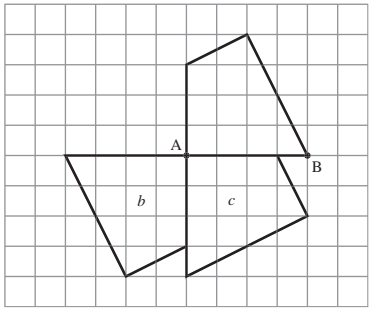
1



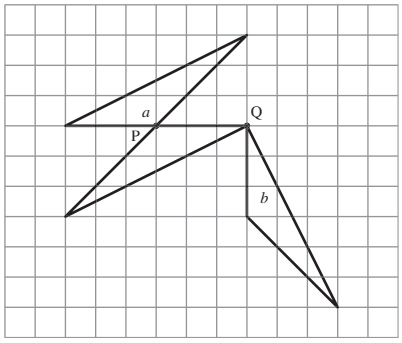
2



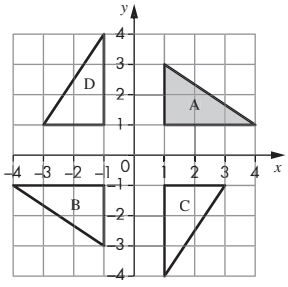
3



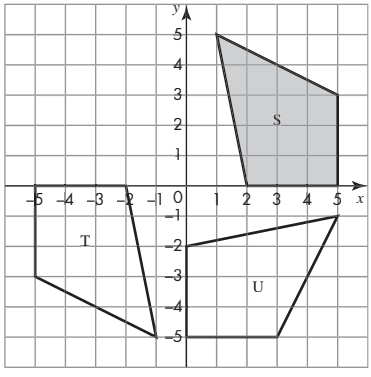
4



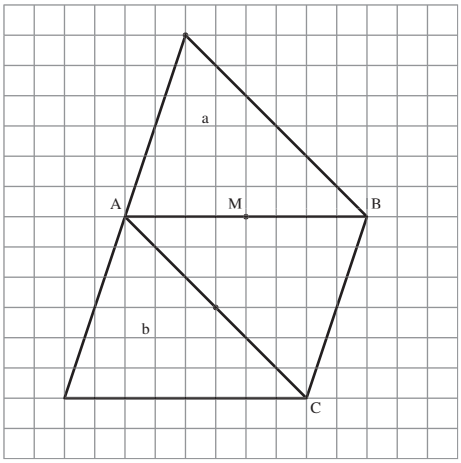
5



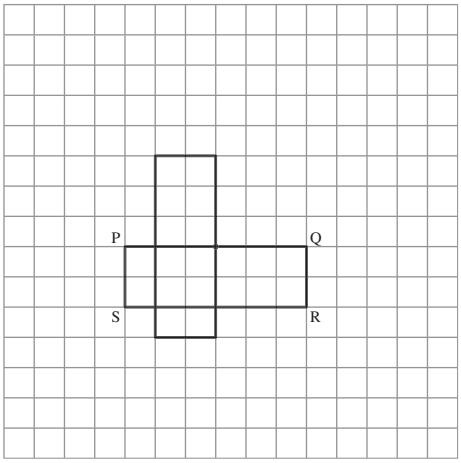
6



7

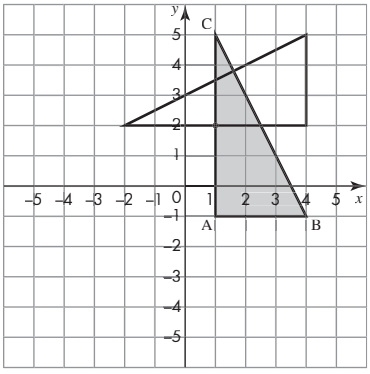


8



9 **a** (1, 2)

b

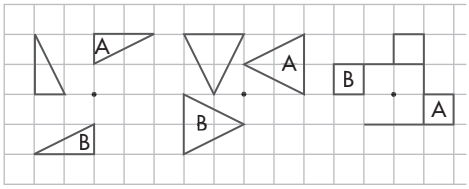


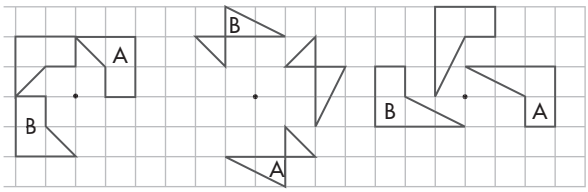
10 **a** (5, 0) **b** (3, 0) **c** (7, 0)

29.5 Rotations: 2

Exercise 29E

1 **a**

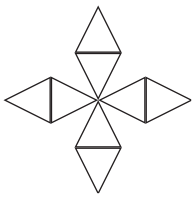




b **i** Rotation 90° anticlockwise

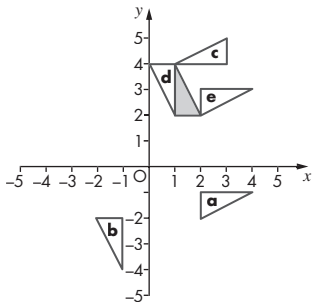
ii Rotation 180°

2



3 Possible answer: If ABCD is the centre square, rotate about A 90° anticlockwise, rotate about new B 180°, now rotate about new C 180°, and finally rotate about new D 180°.

4



5 **a** (4,5) 180°

b (5,5) 90° anticlockwise

c (3,3) 180°

d (3,5) 90° clockwise

6 **a** E **b** H

7 Show by drawing a shape or use the fact that (a, b) becomes $(a, -b)$ after reflection in the x-axis, and $(a, -b)$ becomes $(-a, -b)$ after reflection in the y-axis, which is equivalent to a single rotation of 180°.

8 **a** (3,0) **b** (0,0) **c** (6,0)

9 **a** (0, –1.5) 180° **b** (–0.5, –1.5) 90° clockwise

c (–3,5,2.5) 90° anti clockwise

d (0.5, 2) 180°

10 Show by drawing a shape or use the fact that (a, b) becomes (b, a) after reflection in the line $y = x$, and (b, a) becomes $(-a, -b)$ after reflection in the line $y = -x$, which is equivalent to a single rotation of 180°.

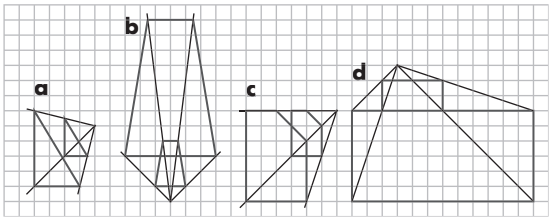
11 Rotation 90° anticlockwise about (3, –2).

12 **a** $y = x$ **b** (1, 1) **c** (6, 6) **d** not possible

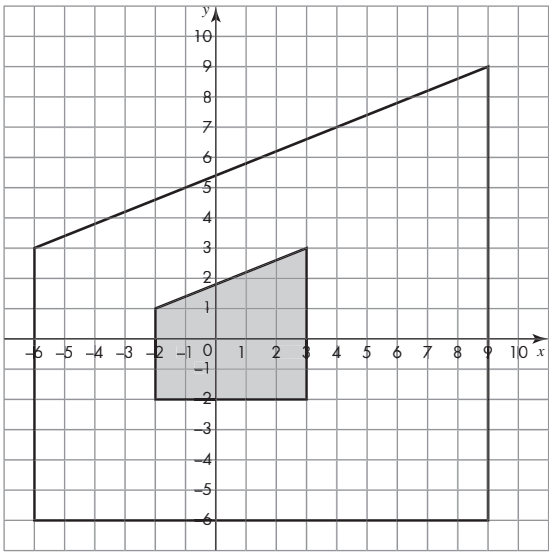
29.6 Enlargements: 1

Exercise 29F

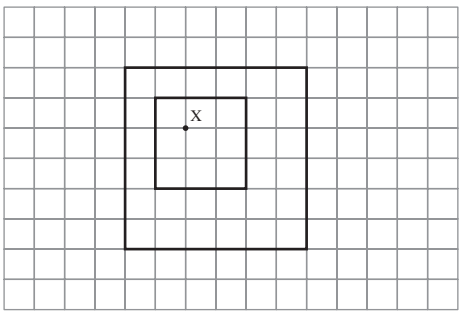
1



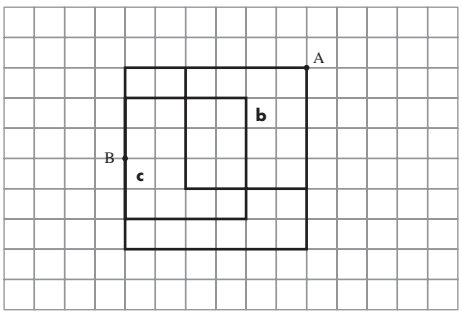
2



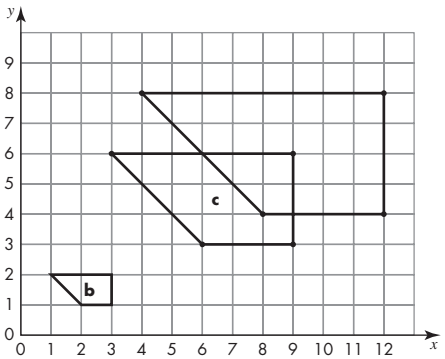
3



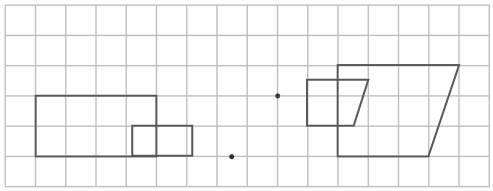
4



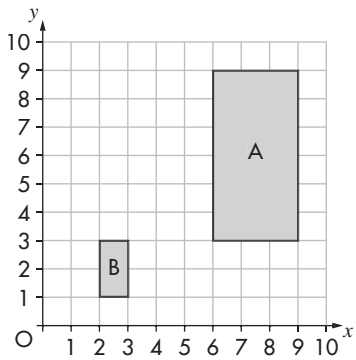
5



6

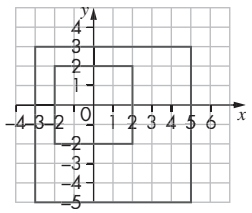


7 a

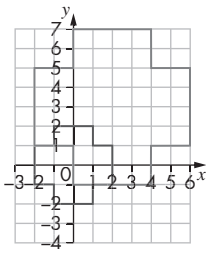


b 3 : 1 **c** 3 : 1 **d** 9 : 1

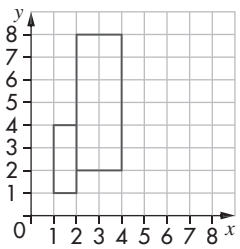
8 a



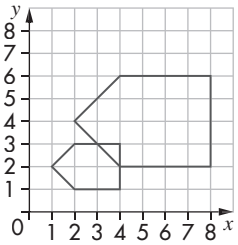
b



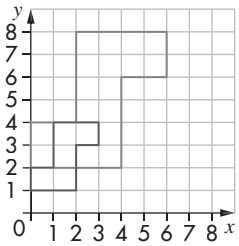
9 a



b



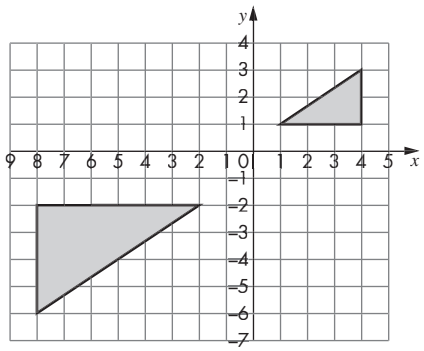
c



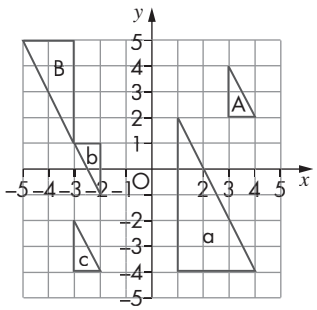
29.7 Enlargements: 2

Exercise 29G

1



2 **a–c**



d Scale factor $-\frac{1}{2}$, centre (1, 3)

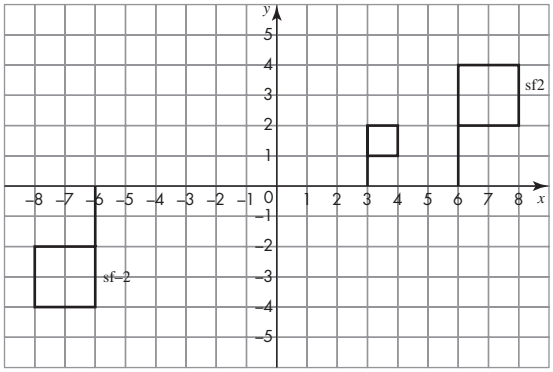
e Scale factor –2, centre (1, 3)

f Scale factor –1, centre (–2.5, –1.5)

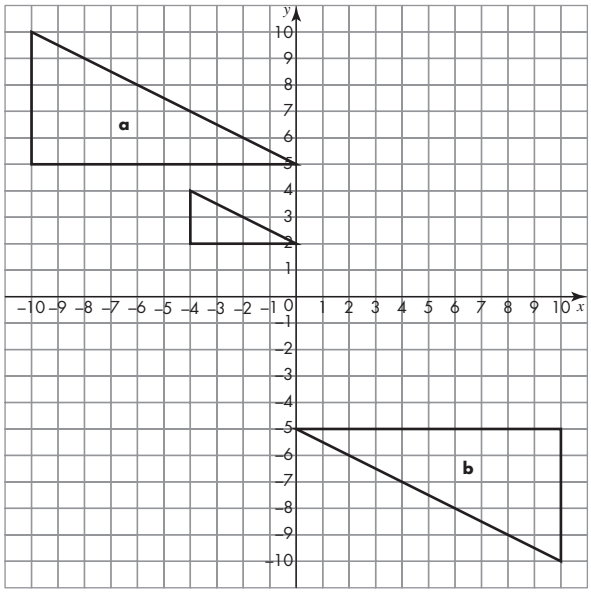
g Scale factor –1, centre (–2.5, –1.5)

h Same centres, and the scale factors are reciprocals of each other

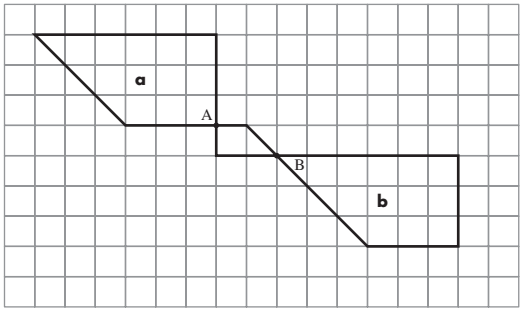
3



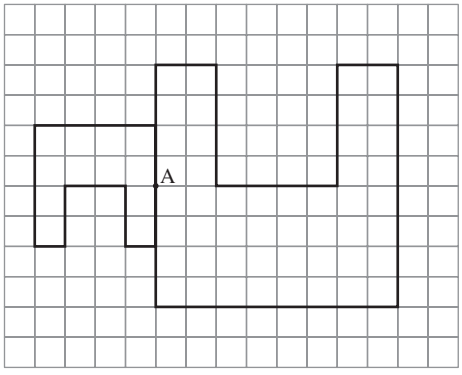
4



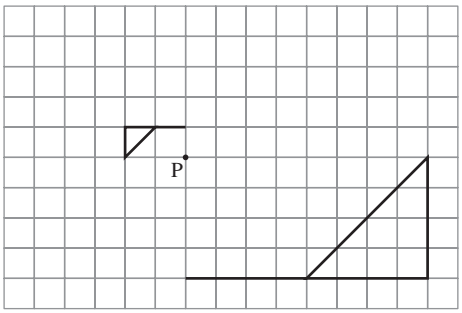
5



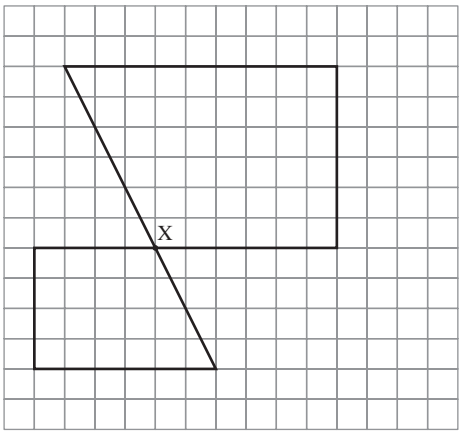
6



7



8



9 **a** Enlargement, centre the origin, scale factor –1.5 or $-\frac{3}{2}$.

b Enlargement, centre the origin, scale factor $-\frac{2}{3}$.

10 Enlargement, scale factor –2, about (1, 3)

11 **a** 9.6 cm **b** 25 : 1

29.8 Combined transformations

Exercise 29H

1 (–4, –3)

2 **a** (–5, 2) **b** Reflection in y-axis

3 A: translation $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$,

B: reflection in y-axis,

C: rotation 90° clockwise about (0, 0),

D: reflection in $x = 3$,

E: reflection in $y = 4$,

F: enlargement by scale factor 2, centre (0, 1)

4 **a** T_1 to T_2: rotation 90° clockwise about (0, 0)

b T_1 to T_6: rotation 90° anticlockwise about (0, 0)

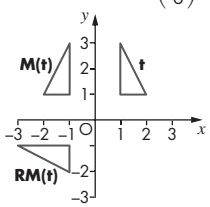
c T_2 to T_3: translation $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$

d T_6 to T_2: rotation 180° about (0, 0)

e T_6 to T_5: reflection in y-axis

f T_5 to T_4: translation $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$

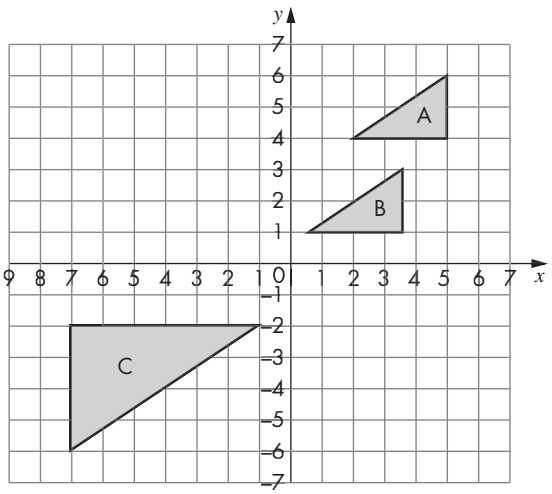
5 **a–c**



d Reflection in the line $y = -x$

6 Reflection in x-axis, translation $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$,rotation 90° clockwise clockwise about (0, 0)

7



b Enlargement, scale factor $-\frac{1}{2}$, centre (1, 2)

Answers to Chapter 30

30.1 Introduction to vectors

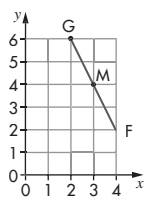
Exercise 30A

1 **a** **i** $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ **ii** $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ **iii** $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

iv $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$ **v** $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$ **vi** $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$

b Both are $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$. D is the midpoint of AC.

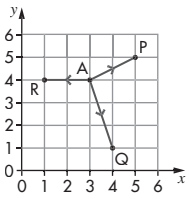
2 **a**



b **i** $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$ **ii** $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$ **iii** $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

iv $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ **v** $\begin{pmatrix} -2 \\ -6 \end{pmatrix}$

3



4 The diagrams should show the following vectors:

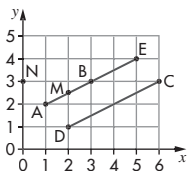
a $\begin{pmatrix}3\\-1\end{pmatrix}$ **b** $\begin{pmatrix}-1\\4\end{pmatrix}$ **c** $\begin{pmatrix}1\\7\end{pmatrix}$

d $\begin{pmatrix}3\\-1\end{pmatrix}$ **e** $\begin{pmatrix}-1\\-7\end{pmatrix}$ **f** $\begin{pmatrix}2\\-8\end{pmatrix}$

5 **a** $\begin{pmatrix}-1\\2\end{pmatrix}$ **b** $\begin{pmatrix}6\\12\end{pmatrix}$ **c** $\begin{pmatrix}-5\\-6\end{pmatrix}$

d $\begin{pmatrix}5\\6\end{pmatrix}$ **e** $\begin{pmatrix}-12\\-8\end{pmatrix}$ **f** $\begin{pmatrix}1\\6\end{pmatrix}$

6 **a–d**



e k is 4.

30.2 Using vectors

Exercise 30B

1 **a** Any three, of: $\overrightarrow{AC}$, $\overrightarrow{CF}$, $\overrightarrow{BD}$, $\overrightarrow{DG}$, $\overrightarrow{GI}$, $\overrightarrow{EH}$, $\overrightarrow{HJ}$, $\overrightarrow{JK}$

b Any three of: $\overrightarrow{BE}$, $\overrightarrow{AD}$, $\overrightarrow{DH}$, $\overrightarrow{CG}$, $\overrightarrow{GJ}$, $\overrightarrow{FI}$, $\overrightarrow{IK}$

c Any three of: $\overrightarrow{AO}$, $\overrightarrow{CA}$, $\overrightarrow{FC}$, $\overrightarrow{IG}$, $\overrightarrow{GD}$, $\overrightarrow{DB}$, $\overrightarrow{KJ}$, $\overrightarrow{JH}$, $\overrightarrow{HE}$

d Any three of: $\overrightarrow{BO}$, $\overrightarrow{EB}$, $\overrightarrow{HD}$, $\overrightarrow{DA}$, $\overrightarrow{JG}$, $\overrightarrow{GC}$, $\overrightarrow{KI}$, $\overrightarrow{IF}$

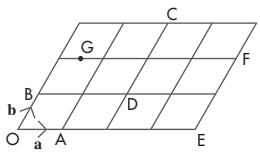
2 **a** $2\mathbf{a}$ **b** $2\mathbf{b}$ **c** $3\mathbf{a}+2\mathbf{b}$ **d** $\mathbf{a}+2\mathbf{b}$

e $\mathbf{a}+\mathbf{b}$ **f** $2\mathbf{a}+2\mathbf{b}$ **g** $3\mathbf{a}+\mathbf{b}$

3 $\overrightarrow{AI}$, $\overrightarrow{BJ}$, $\overrightarrow{DK}$

4 $\overrightarrow{OF}$, $\overrightarrow{BI}$, $\overrightarrow{EK}$

5



6 **a** $\mathbf{a}-\mathbf{b}$ **b** $3\mathbf{a}-\mathbf{b}$ **c** $2\mathbf{a}-\mathbf{b}$ **d** $\mathbf{a}-\mathbf{b}$

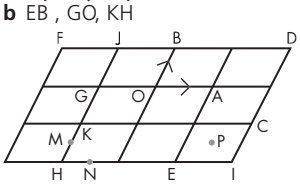
e $\mathbf{a}+\mathbf{b}$ **f** $-\mathbf{a}-\mathbf{b}$ **g** $2\mathbf{a}-\mathbf{b}$ **h** $-\mathbf{a}-2\mathbf{b}$

i $\mathbf{a}+2\mathbf{b}$ **j** $-\mathbf{a}+\mathbf{b}$ **k** $2\mathbf{a}-2\mathbf{b}$ **l** $\mathbf{a}-2\mathbf{b}$

7 **a** $\overrightarrow{BJ}$, $\overrightarrow{CK}$

b $\overrightarrow{EB}$, $\overrightarrow{GO}$, $\overrightarrow{KH}$

8



9 **a** **i** $3\mathbf{a}+2\mathbf{b}$ **ii** $3\mathbf{a}+\mathbf{b}$

iii $2\mathbf{a}-\mathbf{b}$ **iv** $2\mathbf{b}-2\mathbf{a}$

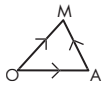
b $\overrightarrow{DG}$ and $\overrightarrow{BC}$

10 **a** $2\mathbf{a}+\mathbf{b}$ **b** $2\mathbf{b}-\mathbf{a}$ **c** $\mathbf{a}+1.5\mathbf{b}$

d $0.5\mathbf{a}+0.5\mathbf{b}$ **e** $1.5\mathbf{a}+1.5\mathbf{b}$ **f** $1.5\mathbf{a}-0.5\mathbf{b}$

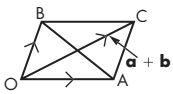
11 **a** **i** $-\mathbf{a}+\mathbf{b}$ **ii** $\frac{1}{2}(-\mathbf{a}+\mathbf{b})$

iii



iv $\frac{1}{2}\mathbf{a}+\frac{1}{2}\mathbf{b}$

b



c M is midpoint of parallelogram of which OA and OB are two sides.

12 **a** **i** $-\mathbf{a}+\mathbf{b}$ **ii** $\frac{1}{3}(-\mathbf{a}+\mathbf{b})$ **iii** $\frac{2}{3}\mathbf{a}+\frac{1}{3}\mathbf{b}$

b $\frac{3}{4}\mathbf{a}+\frac{1}{4}\mathbf{b}$

13 **a** **i** $\frac{2}{3}\mathbf{b}$ **ii** $\frac{1}{2}\mathbf{a}+\frac{1}{2}\mathbf{b}$ **iii** $-\frac{2}{3}\mathbf{b}$

b $\frac{1}{2}\mathbf{a}-\frac{1}{6}\mathbf{b}$

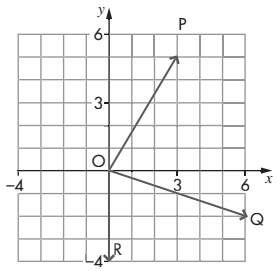
c $\overrightarrow{DE}=\overrightarrow{DO}+\overrightarrow{OE}=\frac{3}{2}\mathbf{a}-\frac{1}{2}\mathbf{b}$

d $\overrightarrow{DE}$ parallel to $\overrightarrow{CD}$ = (multiple of $\overrightarrow{CD}$) and D is a common point

30.3 The magnitude of a vector

Exercise 30C

1 **a**



b $\sqrt{34}$; $\sqrt{40}$; 4

c $\sqrt{58}$

d $\sqrt{40}$

2 **a** 10 and 13 **b** $\begin{pmatrix}11\\-4\end{pmatrix}$ **c** $\sqrt{137}$

d No. 10 + 13 does not equal $\sqrt{137}$

e $\sqrt{401}$ **f** $\sqrt{401}$

g Yes. They are vectors in opposite directions but the same length.

3 **a** 10, 10, 10

b Because they are all the same distance from A. The radius is 10.

4 **a** $\sqrt{17}$ **b** $\sqrt{261}$ **c** 13 **d** 10

Answers to Chapter 31

31.1 Frequency tables

Exercise 31A

1 **a**

Goals	0	1	2	3
Frequency	6	8	4	2

b 1 goal

c 22

2 **a**

Temperature (°C)	14–16	17–19	20–22	23–25	26–28
Frequency	5	10	8	5	2

b 17–19° C

c Getting warmer in the first half and then getting cooler towards the end.

3 **a**

Score	1	2	3	4	5	6
Frequency	5	6	6	6	3	4

b 30

c Yes, frequencies are similar.

4 **a**

Height (cm)	151–155	156–160	161–165	166–170
Frequency	2	5	5	7
Height (cm)	171–175	176–180	181–185	186–190
Frequency	5	4	3	1

b 166–170 cm

c student's survey results

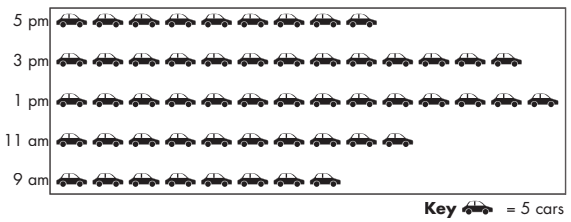
5 Various answers such as 1–10, 11–20, etc. or 1–20, 21–40, 41–60

6 The ages 20 and 25 are in two different groups.

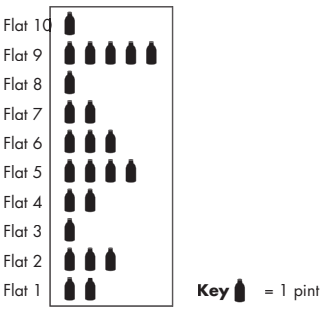
31.2 Pictograms

Exercise 31B

1



2



3 **a** May 10 h, Jun 12 h, Jul 12 h, Aug 12 h, Sep 10 h

b Visual impact, easy to understand.

4 **a** Simon **b** $165

c Difficult to show fractions of a symbol.

5 **a** **i** 12 **ii** 6 **iii** 13

b Check students' pictograms.

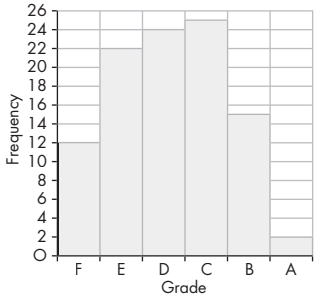
c 63

31.3 Bar charts

Exercise 31C

1 **a** Swimming **b** 74

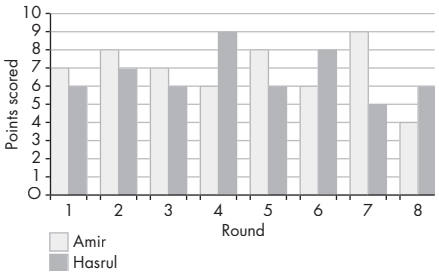
2 **a**



b $\frac{40}{100} = \frac{2}{5}$

c Easier to read the exact frequency.

3 **a**

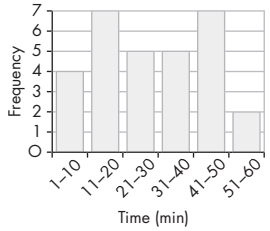


b Amir got more points overall, but Hasrul was more consistent.

4 **a**

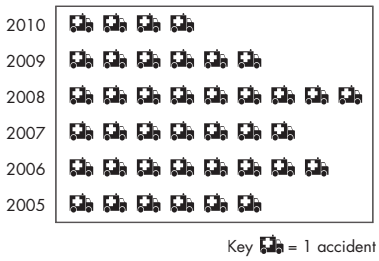
Time (min)	1–10	11–20	21–30	31–40	41–50	51–60
Frequency	4	7	5	5	7	2

b

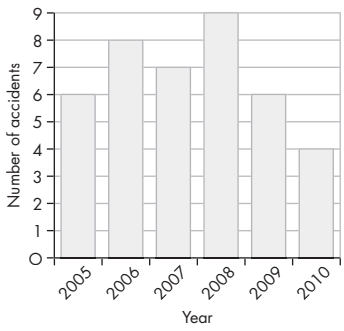


c For example: Some live close to the school. Some live a good distance away and probably travel to school by bus.

5 **a**



b



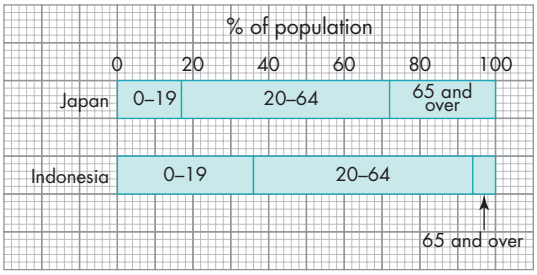
c Use the pictogram because an appropriate symbol makes more impact.

6 Yes. If you double the minimum temperature each time, it is very close to the maximum temperature.

7 **a** Tennis 12, Badminton 16, Volleyball 21, Squash 11

b Volleyball was the most popular sport.

8 **a**

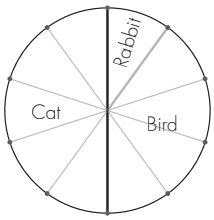


b Japan had a much smaller percentage of people under 20, and a much greater percentage of people who were 65 and over.

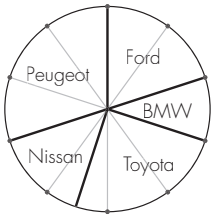
31.4 Pie charts

Exercise 31D

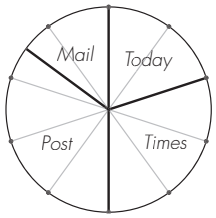
1 **a**



b



c



2 Pie charts with following angles:

a 36°, 90°, 126°, 81°, 27°

b 168°, 52°, 100°, 40°

3 Pie chart with these angles: 60°, 165°, 45°, 15°, 75°

4 **a** 36

b Pie chart with these angles: 50°, 50°, 80°, 60°, 60°, 40°, 20°

c student's bar chart

d Bar chart, because easier to make comparisons.

5 **a** Pie charts with these angles: 124°, 132°, 76°, 28°

b Split of total data seen at a glance.

6 **a** 55° **b** 22

7 **a** Pie charts with these angles: Strings: 36°, 118°, 126°, 72°, 8° Brass: 82°, 118°, 98°, 39°, 23°

b Overall, the Strings candidates did better, as a smaller proportion failed. A higher proportion of Brass candidates scored very good or excellent.

8 $\frac{1}{9}$

9 **a** Accept any valid comment that compares the two schools, such as:

School A had a greater percentage of students attaining the top 10 marks than School B

12.5% of School B obtained 30 or less marks: this was half the percentage of School A's results etc.

Reject answers that refer to numbers of students, e.g. more students got marks in the range 61–90 at School B

b Answers could include:

- the actual numbers of students are unknown
- the size of the pie chart can be misleading.

31.5 Scatter diagrams

Exercise 31E

1 **a** No correlation

b Positive correlation

2 **a** No relationship between temperature and speed of cars.

b As people get older, they have more money in the bank.

3 **a** and **b** student's scatter diagram and line of best fit.

c about 20 cm/s

d about 35 cm

4 **a** student's scatter diagram.

b Yes, usually (good correlation).

5 **a** and **b** Student's scatter diagram and line of best fit.

c Sitara

d about 90

e about 55

6 **a** student's scatter diagram.

b no, because there is no correlation.

7 **a** and **b** Student's scatter diagram and line of best fit.

c about 2.4 km

d 8 minutes

8 23 kilometres/hour

9 Points showing a line of best fit sloping down from top left to bottom right.

31.6 Histograms

Exercise 31F

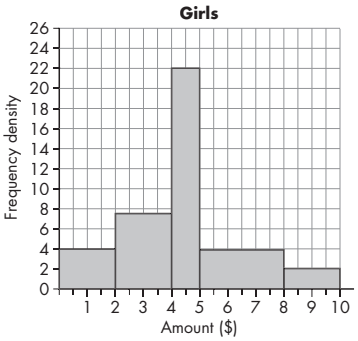
1 The respective frequency densities on which each histogram should be based are:

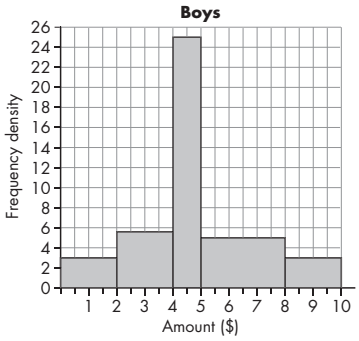
a 2.5, 6.5, 6, 2, 1, 1.5 **b** 4, 27, 15, 3

c 17, 18, 12, 6.67 **d** 0.4, 1.2, 2.8, 1

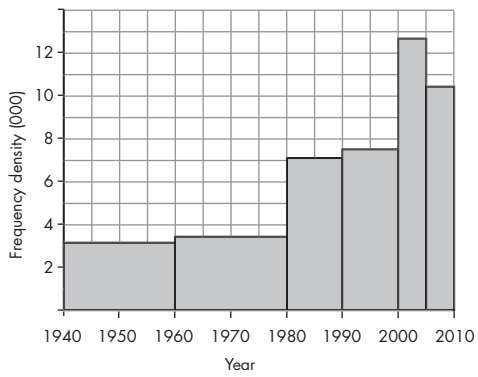
e 9, 21, 13.5, 9

2





3



4 **a** 775 **b** 400

5 **a**

Age, y (years)	$9 < y \leqslant 10$	$10 < y \leqslant 12$	$12 < y \leqslant 14$
Frequency	4	12	8
Age, y (years)	$14 < y \leqslant 17$	$17 < y \leqslant 19$	$19 < y \leqslant 20$
Frequency	9	5	1

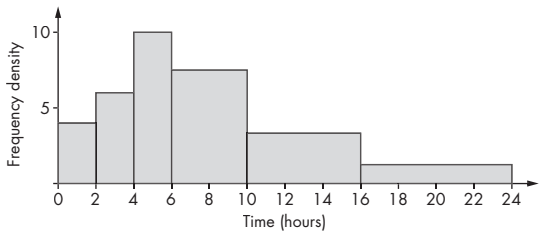
b

Temperature, t (°C)	$10 < t \leqslant 11$	$11 < t \leqslant 12$	$12 < t \leqslant 14$
Frequency	15	15	50
Temperature, t (°C)	$14 < t \leqslant 16$	$16 < t \leqslant 19$	$19 < t \leqslant 21$
Frequency	40	45	15

c

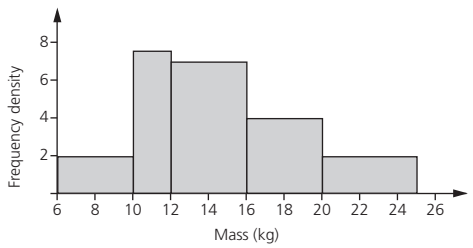
Mass, m (kg)	$50 < m \leqslant 70$	$70 < m \leqslant 90$	$90 < m \leqslant 100$
Frequency	160	200	120
Mass, m (kg)	$100 < m \leqslant 120$	$120 < m \leqslant 170$	
Frequency	120	200	

6 **a**



b 45

7 **a**



b 33 plants

8 **a**

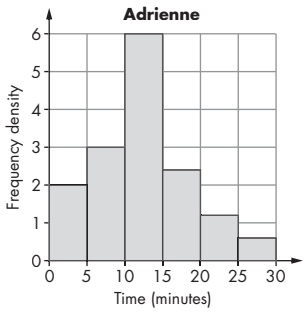
Speed, v (mph)	$0 < v \leqslant 40$	$40 < v \leqslant 50$	$50 < v \leqslant 60$
Frequency	80	10	40
Speed, v (mph)	$60 < v \leqslant 70$	$70 < v \leqslant 80$	$80 < v \leqslant 100$
Frequency	110	60	60

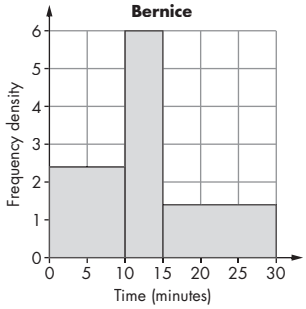
b 360

9 **a** 80

b 31.25%

10 a





b student's own description

Answers to Chapter 32

32.1 The mode

Exercise 32A

1 **a** 4 **b** 48 **c** −1
d $\frac{1}{4}$ **e** no mode **f** 3.21

2 **a** red **b** Sun **c** β **d** ★

3 **a** 32 **b** 6 **c** no
d no; boys generally take larger shoe sizes

4 **a** 5
b no; more than half the form got a higher mark

5 The mode will be the most popular item or brand sold in a shop.

6 **a** 28
b **i** brown **ii** blue **iii** brown
c Both students had blue eyes.

7 **a** May lose count.
b Put in a table, or arrange in order.
c 4

32.2 The median

Exercise 32B

1 **a** 5 **b** 33 **c** $7\frac{1}{2}$ **d** 24
e $8\frac{1}{2}$ **f** 0 **g** 5.25

2 **a** \$2.20 **b** \$2.25
c median, because it is the central value

3 **a** 5
b **i** 15 **ii** 215 **iii** 10 **iv** 10

4 **a** 13, Ella **b** 162 cm, Pat **c** 40 kg, Elisa
d Ella, because she is closest to the 3 medians

5 **a** 12 **b** 14

6 Answers will vary

7 12, 14, 14, 16, 20, 22, 24

8 **a** Possible answer: 11, 15, 21, 21 (one below or equal to 12 and three above or equal)
b Any four numbers higher than or equal to 12, and any two lower or equal
c Eight, all 4 or under

9 A median of \$8 does not take into account the huge value of the \$3000 so is in no way representative.

32.3 The mean

Exercise 32C

1 **a** 6 **b** 24 **c** 45 **d** 1.57 **e** 2

2 **a** 55.1 **b** 324.7 **c** 58.5 **d** 44.9 **e** 2.3

3 **a** 61 **b** 60 **c** 59 **d** Badru **e** 2

4 42 min

5 **a** \$200 **b** \$260 **c** \$278
d Median, because the extreme value of \$480 is not taken into account

6 **a** 35 **b** 36

7 **a** 6
b 16; all the numbers and the mean are 10 more than those in part **a**
c **i** 56 **ii** 106 **iii** 7

8 Possible answers: Speed – Kath, James, John, Joseph; Roberts – Frank, James, Helen, Evie. Other answers are possible.

9 36

10 24

32.4 The range

Exercise 32D

1 **a** 7 **b** 26 **c** 5 **d** 2.4 **e** 7

2 **a** 5°, 3°, 2°, 7°, 3°
b Variable weather over England

3 **a** \$31, \$28, \$33
b \$8, \$14, \$4
c Not particularly consistent

4 **a** 82 and 83
b 20 and 12
c Fay, because her scores are more consistent

5 **a** 5 min and 4 min
b 9 min and 13 min
c Number 50, because times are more consistent

6 **a** Isaac, Oliver, Evrim, Chloe, Lilla, Badru and Isambard
b 70 cm to 92 cm

7 **a** Teachers because they have a high mean and students could not have a range of 20.
b Year 11 students as the mean is 15–16 and the range is 1.

32.5 Which average to use

Exercise 32E

1 **a** **i** 29 **ii** 28 **iii** 27.1
b 14

2 **a** **i** Mode 3, median 4, mean 5
ii 6, 7, $7\frac{1}{2}$
iii 4, 6, 8
b **i** Mean: balanced data
ii Mode: 6 appears five times
iii Median: 28 is an extreme value

3 **a** Mode 73, median 76, mean 80
b The mean, because it is the highest average

4 **a** 150 **b** 20

5 **a** Mean **b** Median
c Mode **d** Median
e Mode **f** Mean

6 No. Mode is 31, median is 31, and mean is 31½.

7 **a** Median **b** Mode **c** Mean

8 Tom mean, David median, Mohamed mode

9 Possible answers: **a** 1, 6, 6, 6, 6 **b** 2, 5, 5, 6, 7

10 Boss chose the mean while worker chose the mode.

11 11.6

12 52.7 kg

32.6 Stem-and-leaf diagrams

Exercise 32F

1 **a** 40 **b** 75 marks **c** 43 marks **d** 71 marks
e You know that half the students got more marks than the median and half got fewer. The mode does not have such a clear use.

2 **a** 18 runners **b** 26.7 s **c** 4.9 s

3 **a** 6 people **b** 35 minutes **c** 70 minutes

4 **a**

2	8 9
3	4 5 6 8 8 9
4	1 1 3 3 3 8 8

b 43 cm **c** 39 cm **d** 20 cm

5 **a**

0	2 8 9 9 9
1	2 3 7 7 8
2	0 1 2 3

b 9 messages **c** 15 messages

6 **a**

0	7 8 9 9
1	0 2 3 4 5 8 8 9 9
2	0 3 4 4 6 8
3	1

key 2 | 3 = 23

b 18 **c** 24

7

	Men	Women
Number of people	41	34
Range of ages	42	33
Median age	43 years	32

8 **a** 8 children

b

	Girls	Boys
Number of children	25	49
Median height	148 cm	146 cm
Range of heights	40 cm	45 cm

c **i** 2 cm more **ii** 19.28 cm less

32.7 Using frequency tables

Exercise 32G

1 **a** **i** 7 **ii** 6 **iii** 6.4
b **i** 4 **ii** 4 **iii** 3.7
c **i** 8 **ii** 8.5 **iii** 8.2
d **i** 0 **ii** 0 **iii** 0.3

2 **a** 668 **b** 1.9 **c** 0 **d** 328

3 **a** 2.2, 1.7, 1.3 **b** Better dental care

4 **a** 0 **b** 0.96

5 **a** 7 **b** 6.5 **c** 6.5

6 **a** 1 **b** 1 **c** 0.98

7 **a** Roger 5, Brian 4
b Roger 3, Brian 8
c Roger 5, Brian 4
d Roger 5.4, Brian 4.5
e Roger, because he has the smaller range
f Brian, because he has the better mean

8 Possible answers: 3, 4, 15, 3 or 3, 4, 3, 15 …

9 Add up the weeks to see she travelled in 52 weeks of the year, the median is in the 26th and 27th week. Looking at the weeks in order, the 23rd entry is the end of 2 days in a week so the median must be in the 3 days in a week.

32.8 Grouped data

Exercise 32H

1 **a** **i** $30 < x \leqslant 40$ **ii** 29.5
b **i** $0 < y \leqslant 100$ **ii** 158.3
c **i** $5 < z \leqslant 10$ **ii** 9.43
d **i** 7–9 **ii** 8.4 weeks

2 **a** $100 < m \leqslant 120$ g **b** 10 860 g **c** 108.6 g

3 **a** 207 **b** 19–22 cm **c** 20.3 cm

4 **a** 160 **b** 52.6 min **c** modal group
d 65%

5 **a** $175 < h \leqslant 200$ **b** 31% **c** 193.25
d No: mode, mean and median are all less than 200 hours

6 Average price increases: Soundbuy 17.6p, Springfields 18.7p, Setco 18.2p

7 Yes: average distance is 11.7 miles per day.

8 The first 5 and the 10 are the wrong way round.

9 $740

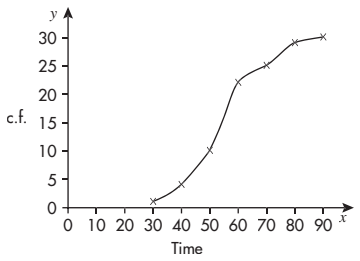
10 As we do not know what numbers are in each group, we cannot say what the median is.

32.9 Cumulative frequency diagrams

Exercise 32I

1 **a** Cumulative frequency 1, 4, 10, 22, 25, 28, 30

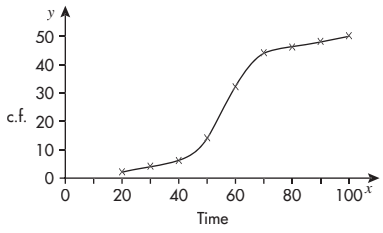
b



c 54 secs, 16 secs

2 **a** Cumulative frequency 1, 3, 5, 14, 31, 44, 47, 49, 50

b

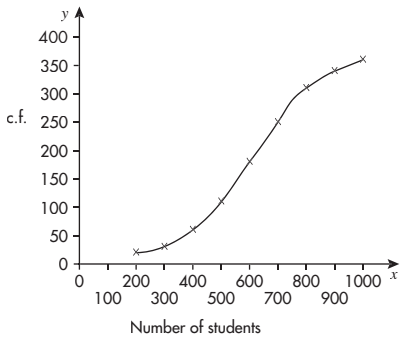


c 56 secs, 17 secs

d Pensioners, median closer to 60 secs

3 **a** Cumulative frequency 12, 30, 63, 113, 176, 250, 314, 349, 360

b



c 605 students, 280 students

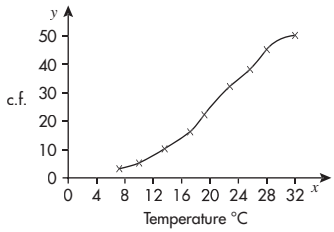
d 46–47 schools

e about 830

f about 550

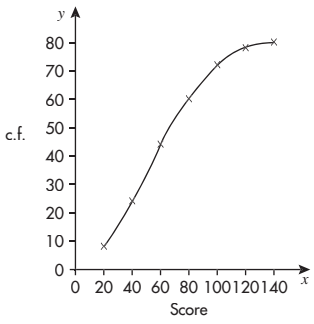
4 **a** Cumulative frequency 2, 5, 10, 16, 22, 31, 39, 45, 50

b



c 20.5°C, 10°C **d** 10.5°C

5 **a**

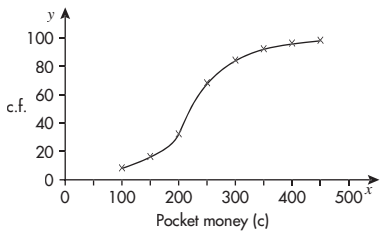


b 56, 43

c about 17.5%

6 **a** Cumulative frequency 6, 16, 36, 64, 82, 93, 98, 100

b



c 225c, 90c

d about 120 cents and about 340 cents

7 **a** Paper A 68, Paper B 57

b Paper A 28, Paper B 18

c Paper B is the harder paper, it has a lower median and a lower upper quartile.

d **i** Paper A 43, Paper B 45 **ii** Paper A 78, Paper B 67

8 **a** about 40% **b** about 6 minutes

9 Find the top 10% on the cumulative frequency scale, read along to the graph and read down to the marks. The mark seen will be the minimum mark needed for this top grade.

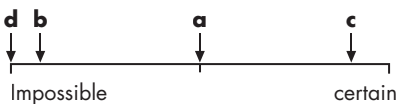
Answers to Chapter 33

33.1 Probability scale

Exercise 33A

1 **a** unlikely **b** unlikely **c** impossible
d very likely **e** even chance

2



e student's own estimate

3 student's own estimate

4 Student to provide own answers.

5 No. What happens today does not depend on what happened yesterday.

33.2 Calculating probabilities

Exercise 33B

1 **a** $\frac{1}{10}$ **b** $\frac{4}{10}$ or $\frac{2}{5}$ **c** $\frac{7}{10}$
d $\frac{1}{2}$ **e** 0

2 **a** $\frac{1}{8}$ **b** $\frac{5}{8}$ **c** $\frac{1}{2}$

3 **a** 0 **b** 1

4 **a** $\frac{1}{10}$ **b** $\frac{1}{2}$ **c** $\frac{2}{5}$ **d** $\frac{1}{5}$ **e** $\frac{2}{5}$

5 **a** $\frac{6}{11}$ **b** $\frac{5}{11}$ **c** $\frac{6}{11}$

6 **a** $\frac{1}{5}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$ **d** $\frac{7}{10}$

7 $\frac{1}{25}$

8 **a** AB, AC, AD, AE, BC, BD, BE, CD, CE, DE
b 1 **c** $\frac{1}{10}$ **d** 6 **e** $\frac{3}{5}$ **f** $\frac{3}{10}$

9 **a** **i** $\frac{12}{25}$ **ii** $\frac{7}{25}$ **iii** $\frac{6}{25}$
b They add up to 1.
c All possible outcomes are mentioned.

10 35%

11 0.5

12 Class U

13 There might not be the same number of boys as girls in the class.

33.3 Probability that an event will not happen

Exercise 33C

1 **a** $\frac{3}{4}$ **b** 0.55 **c** 0.2

2 **a** $\frac{3}{4}$ **b** $\frac{17}{20}$ **c** $\frac{19}{20}$

3 **a** **i** $\frac{1}{4}$ **ii** $\frac{3}{4}$
b **i** $\frac{3}{11}$ **ii** $\frac{8}{11}$

4 Because it might be possible for the game to end in a draw.

33.4 Probability in practice

Exercise 33D

1 **a** 0.2, 0.08, 0.1, 0.105, 0.148, 0.163, 0.1645
b 6 **c** 1 **d** $\frac{1}{6}$ **e** 1000

2 **a** 0.095, 0.135, 0.16, 0.265, 0.345
b 40 **c** No; all numbers should be close to 40.

3 **a** 0.2, 0.25, 0.38, 0.42, 0.385, 0.3974
b 8

4 **a** 6 **b** and **c** Student to provide own answers.

5 **a** Caryl, threw the greatest number of times.
b 0.39, 0.31, 0.17, 0.14
c Yes; all answers should be close to 0.25.

6 The missing top numbers are 4 and 5, the bottom two numbers are both likely to be close to 20.

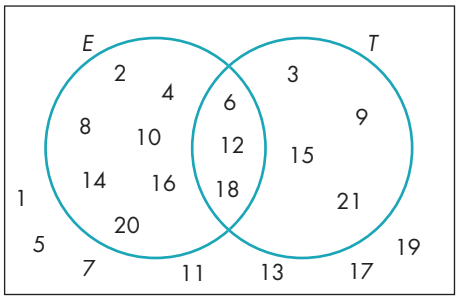
7 Thursday

8 Although he might expect the probability to be close to $\frac{1}{2}$ giving 500 heads, the actual number of heads is unlikely to be exactly 500, but should be close to it.

33.5 Using Venn diagrams

Exercise 33E

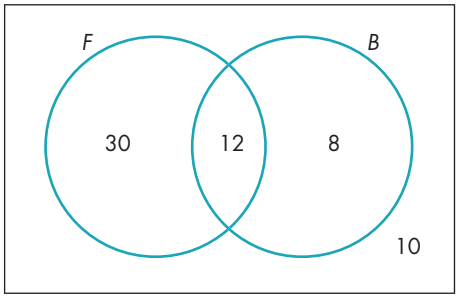
1 **a**



b **i** $\frac{10}{21}$ **ii** $\frac{7}{21} = \frac{1}{3}$ **iii** $\frac{3}{21} = \frac{1}{7}$

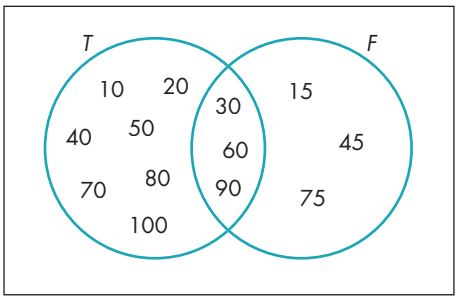
2 **a** $\frac{60}{100} = \frac{3}{5}$ or 0.6 **b** $\frac{35}{100} = \frac{7}{20}$ or 0.35
c $\frac{75}{100} = \frac{3}{4}$ or 0.75 **d** $\frac{25}{100} = \frac{1}{4}$ or 0.25

3 **a**



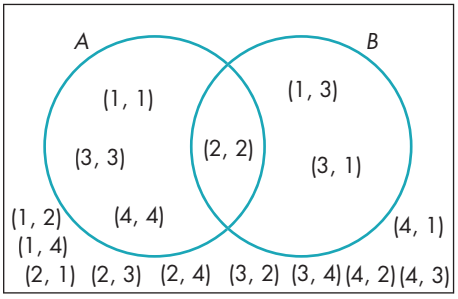
b **i** $\frac{12}{60} = \frac{1}{5}$ or 0.2 **ii** $\frac{42}{60} = \frac{7}{10}$ or 0.7 **c** $\frac{20}{60} = \frac{1}{3}$

4 **a**



b **i** $\frac{6}{100} = \frac{3}{50}$ **ii** $\frac{3}{100}$ **iii** $\frac{87}{100}$

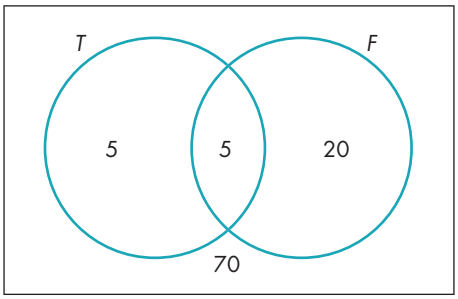
5 **a**



b **i** $\frac{4}{16} = \frac{1}{4}$ **ii** $\frac{3}{16}$ **iii** $\frac{1}{16}$

6 **a** 10

b



c **i** $\frac{10}{100} = \frac{1}{10}$ **ii** $\frac{25}{100} = \frac{1}{4}$ **iii** $\frac{5}{100} = \frac{1}{20}$ **iv** $\frac{70}{100} = \frac{7}{10}$

33.6 Probability notation

Exercise 33F

1 **a** **i** $\frac{1}{4}$ **ii** $\frac{3}{4}$ **iii** $\frac{5}{8}$

iv $\frac{3}{8}$ **v** $\frac{1}{8}$ **vi** $\frac{3}{4}$

b **i** The probability of 6 or more

ii The probability of a multiple of 3 that is less than 6

2 **a** 0.66 **b** 94%

3 **a** $\frac{5}{13}$ **b** $\frac{6}{13}$ **c** $\frac{8}{13}$

d $\frac{7}{13}$ **e** $\frac{2}{13}$ **f** $\frac{9}{13}$

4 **a** $\frac{1}{3}$ **b** $\frac{1}{4}$ **c** $\frac{2}{3}$

d $\frac{3}{4}$ **e** $\frac{1}{12}$ **f** $\frac{1}{2}$

5 **a** 70 students play football.

b There are 90 students in total.

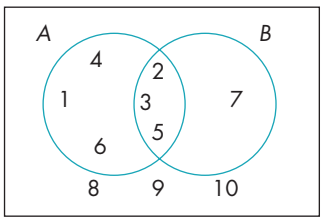
c **i** $\frac{7}{9}$ **ii** $\frac{2}{9}$ **iii** $\frac{1}{2}$

iv $\frac{1}{2}$ **v** $\frac{1}{3}$ **vi** $\frac{17}{18}$

d **i** The probability that the student does not play basketball

ii The probability that the student plays both football and basketball

6 **a**



b **i** $\frac{3}{5}$ **ii** $\frac{2}{5}$ **iii** $\frac{3}{5}$

iv $\frac{3}{10}$ **v** $\frac{7}{10}$ **vi** $\frac{1}{10}$

c **i** The probability of 7 or more

ii The probability of a prime number that is less than 7

33.7 Sample space diagrams

Exercise 33G

1 **a** 7 **b** 2 and 12

c $\frac{1}{36}, \frac{1}{18}, \frac{1}{12}, \frac{1}{9}, \frac{5}{36}, \frac{1}{6}, \frac{5}{36}, \frac{1}{9}, \frac{1}{12}, \frac{1}{18}, \frac{1}{36}$

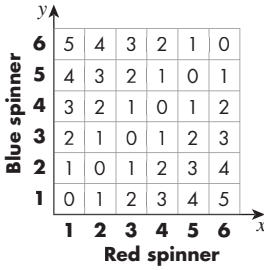
d **i** $\frac{1}{12}$ **ii** $\frac{1}{3}$ **iii** $\frac{1}{2}$ **iv** $\frac{7}{36}$

v $\frac{5}{12}$ **vi** $\frac{5}{18}$

2 **a** $\frac{1}{12}$ **b** $\frac{11}{36}$ **c** $\frac{1}{6}$ **d** $\frac{5}{9}$

3 **a** $\frac{1}{36}$ **b** $\frac{11}{36}$ **c** $\frac{5}{18}$

4



a $\frac{5}{18}$ **b** $\frac{1}{6}$ **c** $\frac{1}{9}$

d 0 **e** $\frac{1}{2}$

5 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$

c $\frac{3}{4}$ **d** $\frac{1}{4}$

6

Second spinner \ First spinner	1	2	3	4	5
5	6	7	8	9	10
4	5	6	7	8	9
3	4	5	6	7	8
2	3	4	5	6	7
1	2	3	4	5	6

a 6

b **i** $\frac{4}{25}$ **ii** $\frac{13}{25}$ **iii** $\frac{1}{5}$ **iv** $\frac{3}{5}$

7 **a**

Second spinner \ First spinner	1	2	3	4	5	6	7	8
8	8	16	24	32	40	48	56	64
7	7	14	21	28	35	42	49	56
6	6	12	18	24	30	36	42	48
5	5	10	15	20	25	30	35	40
4	4	8	12	16	20	24	28	32
3	3	6	9	12	15	18	21	24
2	2	4	6	8	10	12	14	16
1	1	2	3	4	5	6	7	8

b $\frac{8}{64} = \frac{1}{8}$

8 $\frac{7}{36}$: a diagram will help him to see all possible outcomes

33.8 Tree diagrams

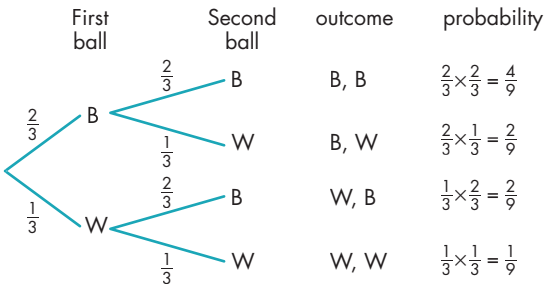
Exercise 33H

1 **a** $\frac{1}{4}$

b $\frac{1}{2}$

c $\frac{3}{4}$

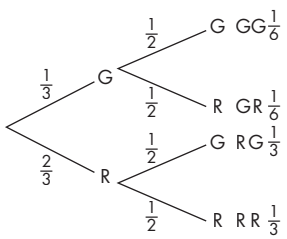
2 **a**



b **i** $\frac{1}{9}$ **ii** $\frac{4}{9}$ **iii** $\frac{8}{9}$

3 **a** $\frac{2}{3}$ **b** $\frac{1}{2}$

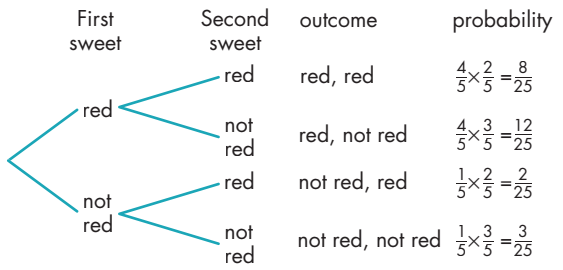
c



d **i** $\frac{1}{6}$ **ii** $\frac{1}{2}$ **iii** $\frac{5}{6}$

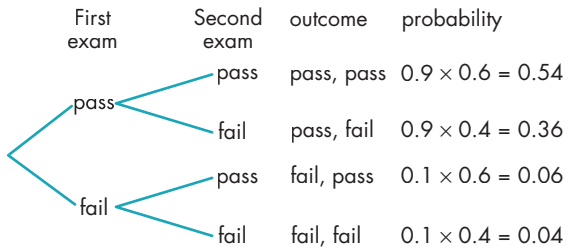
e 15 days

4 **a**



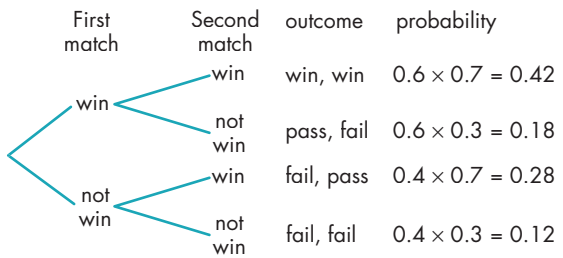
b **i** $\frac{8}{25}$ **ii** $\frac{22}{25}$ **iii** $\frac{3}{25}$

5 **a**



b **i** 0.54 **ii** 0.42

6 **a**



b 0.88

7 **a** 0.09 **b** 0.49 **c** 0.42

8 **a** 0.1 **b** 0.3 **c** 0.55

9 0.53

10 **a** **i** $\frac{5}{13}$ **ii** $\frac{8}{13}$

b **i** $\frac{15}{91}$ **ii** $\frac{4}{13}$

11 **a** $\frac{1}{120}$ **b** $\frac{7}{40}$ **c** $\frac{21}{40}$ **d** $\frac{7}{24}$

12 **a** $\frac{1}{9}$ **b** $\frac{2}{9}$ **c** $\frac{2}{3}$ **d** $\frac{7}{9}$

13 **a** 0.54 **b** 0.38 **c** 0.08 **d** 1

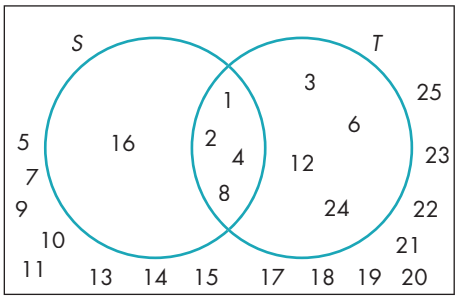
33.9 Conditional probability

Exercise 33I

1 **a** $\frac{55}{100} = \frac{11}{20}$ or 0.55 **b** $\frac{22}{55} = \frac{2}{5}$ or 0.4

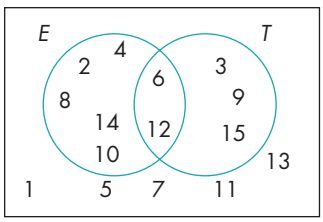
c $\frac{22}{50} = \frac{11}{25}$ or 0.44

2 **a**



b $\frac{4}{25}$ **c** $\frac{4}{8} = \frac{1}{2}$ **d** $\frac{4}{5}$ **e** $\frac{16}{20} = \frac{4}{5}$

3 **a**



b **i** $\frac{7}{15}$ **ii** $\frac{2}{3}$ **iii** $\frac{2}{15}$

c $\frac{2}{7}$

d $\frac{2}{5}$

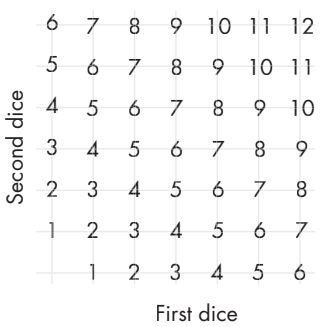
e $\frac{5}{8}$

4 **a** **i** $\frac{13}{24}$ **ii** $\frac{7}{24}$ **iii** $\frac{5}{12}$ **iv** $\frac{1}{6}$

b **i** $\frac{10}{17}$ **ii** $\frac{7}{17}$

c $\frac{4}{11}$

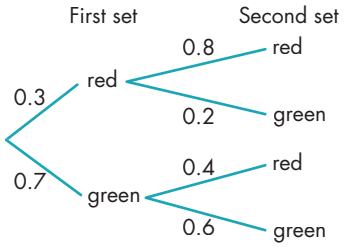
5 **a**



b $\frac{5}{36}$ **c** $\frac{2}{5}$ **d** $\frac{11}{36}$ **e** $\frac{2}{11}$

6 **a** $\frac{6}{16} = \frac{3}{8}$ **b** $\frac{2}{6} = \frac{1}{3}$ **c** $\frac{2}{4} = \frac{1}{2}$

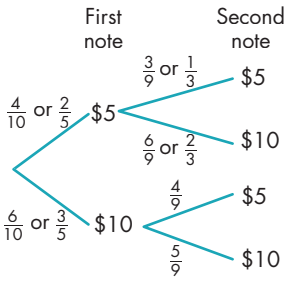
7 **a**



b 0.24 **c** 0.42 **d** 0.34

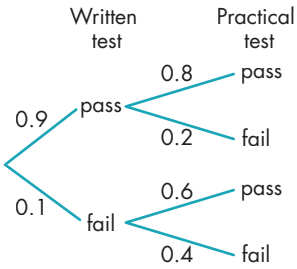
8 **a** $\frac{4}{10}$ or $\frac{2}{5}$ **b** $\frac{3}{9}$ or $\frac{1}{3}$

c



d **i** $\frac{2}{15}$ **ii** $\frac{8}{15}$ **iii** $\frac{1}{3}$

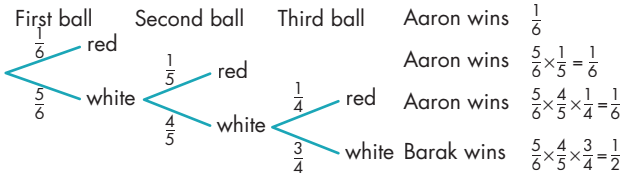
9 **a**



b 0.72 **c** 0.24

10 **a** $\frac{3}{28}$ **b** $\frac{5}{14}$ **c** $\frac{1}{56} + \frac{5}{28} = \frac{11}{56}$

11 The tree diagram looks like this



Aaron wins if the first ball or the second or the third is red.

The probability of this is $\frac{1}{6} + \left(\frac{5}{6} \times \frac{1}{5}\right) + \left(\frac{5}{6} \times \frac{4}{5} \times \frac{1}{4}\right)$

$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

Or: Barak wins if there are 3 white balls and the probability of this is $\frac{5}{6} \times \frac{4}{5} \times \frac{3}{4} = \frac{1}{2}$

Hence the probability that Aaron wins is $1 - \frac{1}{2} = \frac{1}{2}$